Analytical Study of the Large Orbital X-Ray Telescope Imaging System

Contract No. NAS8-29855

FINAL REPORT

by

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August, 1973

(NASA-CR-124429) ANALYTICAL STUDY OF THE LARGE ORBITAL X-RAY TELESCOPE IMAGING SYSTEM Final Report (Montevallo Univ., Ala.) 46 p HC $4.50 CSCL 20F

N73-32338

Unclas

G3/14 15621

Submitted to

National Aeronautics and Space Administration

George C. Marshall Space Flight Center

Marshall Space Flight Center, Alabama

by

UNIVERSITY OF MONTEVALLO

Montevallo, Alabama

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I. INTRODUCTION

This report presents the results of an analytical study of the Large Orbital X-Ray Telescope (LOXT) designed by American Science and Engineering. The LOXT consists of a nested array of four conventional paraboloidal-hyperboloidal x-ray telescopes arranged with a common optical axis and a common focal plane. The composite nested array has a nominal effective focal length of 135.0 inches. The equations of the various mirror surfaces and the numerical values of the parameters in the defining equations are given in the technical specification sheets for the LOXT issued by American Science and Engineering.

The work reported herein was carried out by two faculty members of the Department of Mathematics and Physics at the University of Montevallo: Dr. J. William Foreman, Jr. and Mr. Joseph M. Cardone. Dr. Foreman served as Principal Investigator for the study. This report covers work done during the entire contract period from 1 July, 1973 to 31 August, 1973.

Except for computer graphics, all computer programs were run in the automatic extended precision mode on an XDS-Sigma 5 computer in Wing C of the Astrionics Laboratory at NASA-MSFC. Spot diagrams and line spread functions were plotted by a Calcomp Model 566 digital plotter controlled by an IBM-1130 computer located in Wing B of the Astrionics Laboratory.

The results reported herein were obtained by the general ray trace methods documented in Ref. 1. A more detailed treatment of the analytical approach will be given in the final report for Contract NAS8-30375, which is a continuation of the work begun in the present contract.

II. PROGRAM OUTLINE

The analytical study of the LOXT consisted of five distinct parts: (1) Calculation of the rms and maximum spot diameters in the focal plane as a function of off-axis angle for a point source at infinity, and plotting of representative spot diagrams. These data were obtained for each of the four paraboloidal-hyperboloidal mirror sets individually, and also for the composite system (all four mirrors operating simultaneously).

(2) Calculation of the rms and maximum spot diameters in the plane of best on-axis focus as a function of off-axis angle for a point source at a finite distance of 1000 feet from the LOXT. Data were obtained for each mirror set individually, and for the composite system.

(3) Determination of the field curvature of each paraboloidal-hyperboloidal mirror set, and the field curvature of the composite system, for a point source at infinity.

(4) Calculation of the radial and tangential line spread functions for the composite system in the focal plane at various off-axis angles for a line source at infinity.

(5) Calculation of the radial and tangential line spread functions for the composite system in the plane of best on-axis focus at various off-axis angles for a line source at a finite distance of 1000 feet from the LOXT.

NOTE: In Parts (4) and (5), the line spread functions were calculated at two different x-ray wavelengths, 4 Å and 40 Å. The line spread functions were also calculated assuming 100% reflection at every mirror surface, for comparison with the 4 Å and 40 Å results.

In the ray trace analysis of the LOXT, the origin of the coordinate system was chosen to lie on the optical axis in the focal plane of the telescope, as shown in Fig. 1. The x-axis was chosen to be the optical axis, to conform with the coordinate system used by American Science and Engineering in their specification sheets.

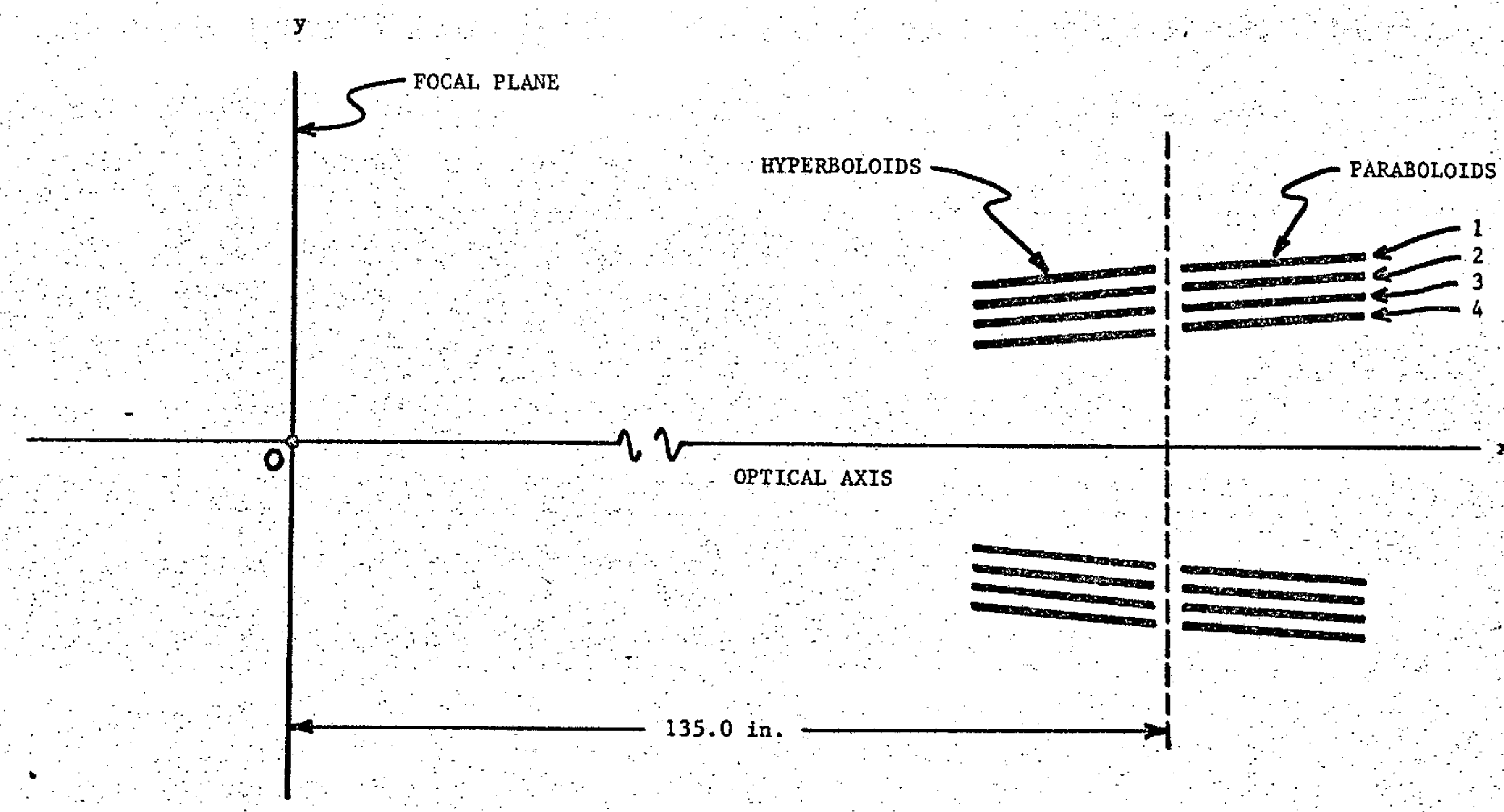


Figure 1. Coordinate system used in the ray trace analysis of the LOXT.

III. RESULTS

In this section, the quantitative results of the analytical study of the LOXT are presented in tables and graphs, each of which is self explanatory.

Maximum and rms spot diameters in the focal plane as a function of off-axis angle for a point source at infinity.

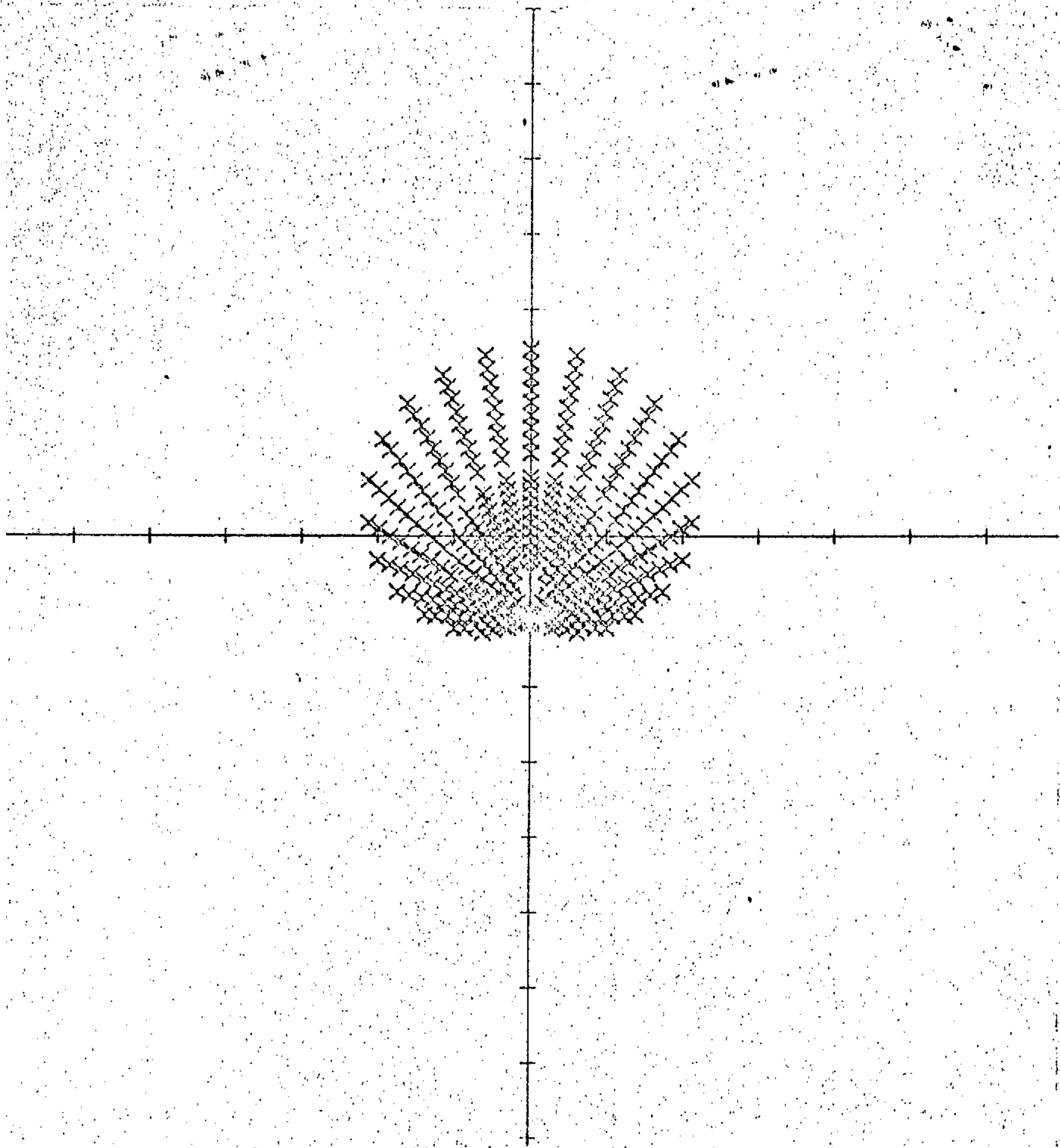
NOTE: 190 rays were traced through each individual mirror to obtain the spot sizes listed in the table below. 760 rays, 190 through each mirror, were used to obtain the tabulated spot sizes for the composite mirror system.

OFF-AXIS ANGLE (arc-minutes)	Mirror No.	RMS Spot Diameter (arc-seconds)	Maximum Spot Diameter (arc-seconds)
0.5	1	0.112147	0.178788
	2	0.090454	0.160892
	3	0.079468	0.152659
	4	0.080983	0.155859
	Comp.	0.120287	0.290426
1.0	1	0.267863	0.496689
	2	0.249443	0.483829
	3	0.259901	0.498130
	4	0.299929	0.548155
	Comp.	0.312344	0.721699
1.5	1	0.491949	0.952448
	2	0.500798	0.967121
	3	0.557542	1.034049
	4	0.665093	1.172775
	Comp.	0.607077	1.293320
2.0	1	0.794427	1.544891
	2	0.849777	1.609202
	3	0.974122	1.758263
	4	1.177061	2.026821
	Comp.	1.013435	2.218702

Location of centers of spots formed by the individual mirrors as a function of off-axis angle, showing misregistration of the spots.

OFF-AXIS ANGLE (arc-minutes)	MIRROR NO.	DISPLACEMENT OF SPOT CENTER FROM ORIGIN (inches)	MISREGISTRATION* (arc-seconds)
0.5	1	0.019737	0.104
	2	0.019709	
	3	0.019687	
	4	0.019669	
1.0	1	0.039474	0.211
	2	0.039418	
	3	0.039373	
	4	0.039336	
1.5	1	0.059209	0.318
	2	0.059125	
	3	0.059056	
	4	0.059001	
2.0	1	0.078943	0.432
	2	0.078830	
	3	0.078737	
	4	0.078660	

* Misregistration is calculated as SPOT DISPLACEMENT(Mirror #1) - SPOT DISPLACEMENT(Mirror #4), expressed in arc-seconds instead of inches.

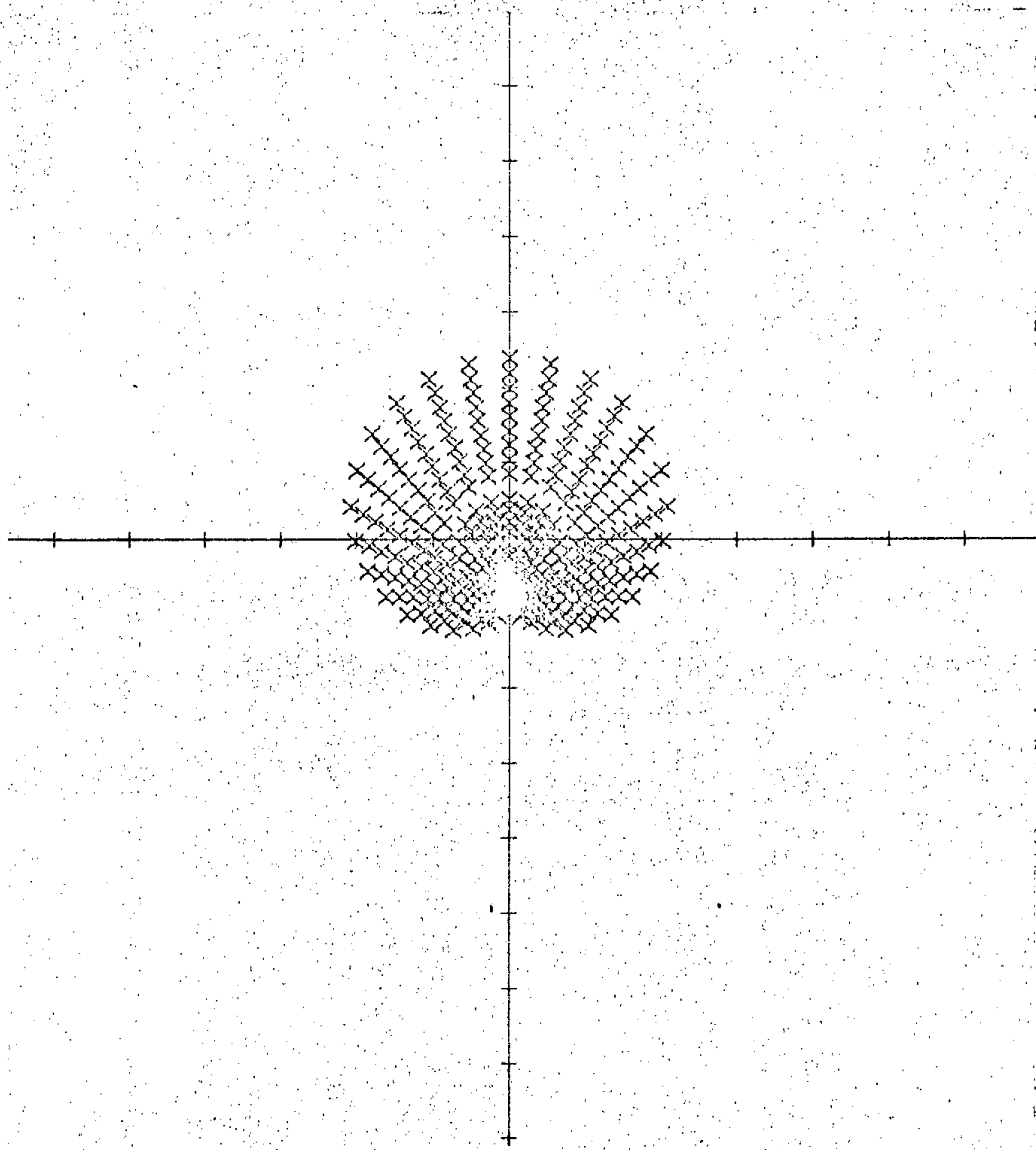


OFF-AXIS ANGLE = 1.0 ARC-MINUTES

MIRROR SYSTEM NO. 1

POINT SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND

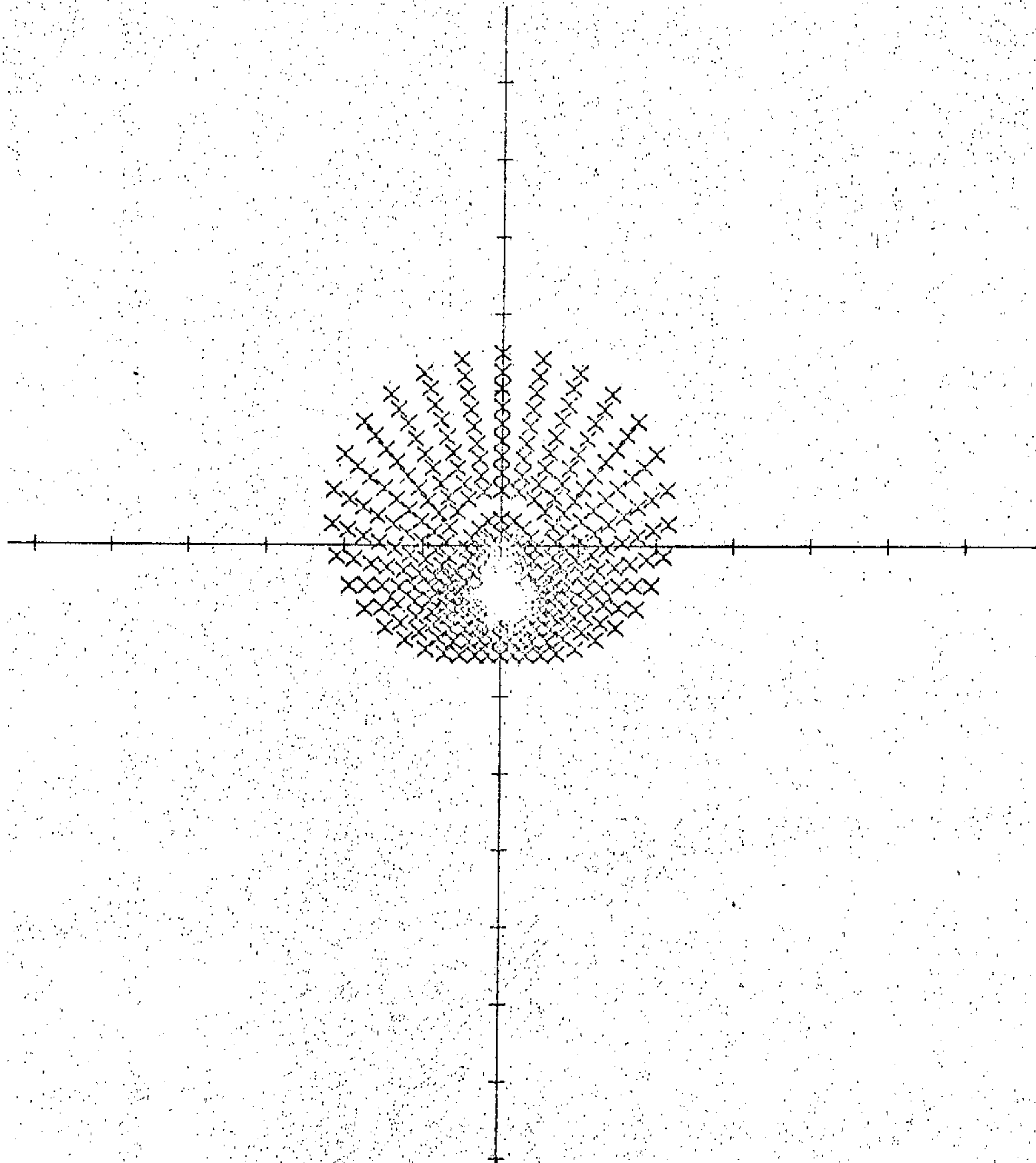


OFF-AXIS ANGLE = 1.0 ARC-MINUTES

MIRROR SYSTEM NO. 2

POINT SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND

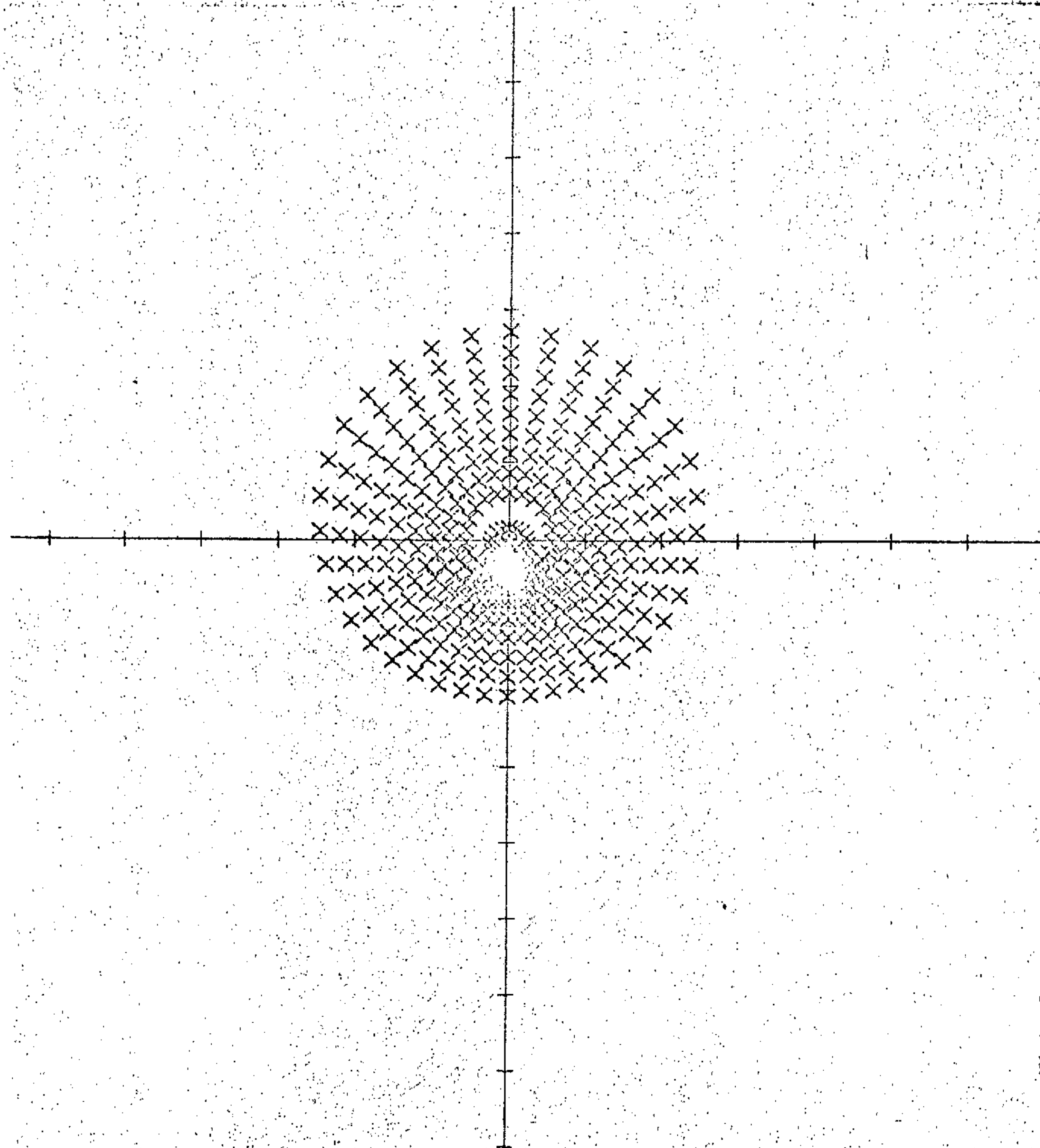


OFF-AXIS ANGLE = 1·0 ARC-MINUTES

MIRROR SYSTEM NO. 3

POINT SOURCE AT INFINITY

ONE SCALE DIVISION = 0·1 ARC-SECOND

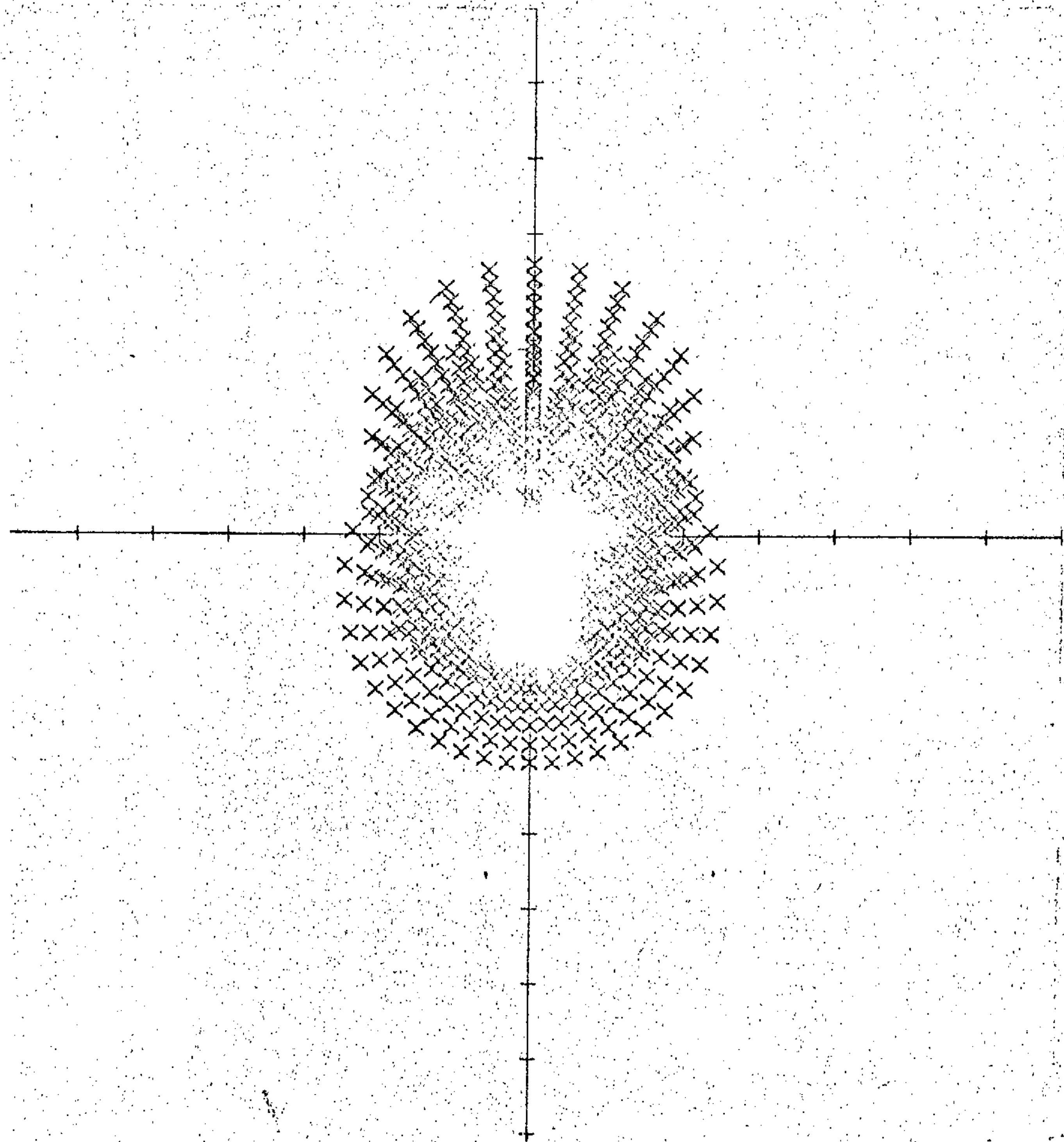


OFF-AXIS ANGLE = 1.0 ARC-MINUTES

MIRROR SYSTEM NO. 4

POINT SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND



OFF-AXIS ANGLE = 1·0 ARC-MINUTES

COMPOSITE MIRROR SYSTEM

POINT SOURCE AT INFINITY

ONE SCALE DIVISION = 0·1 ARC-SECOND

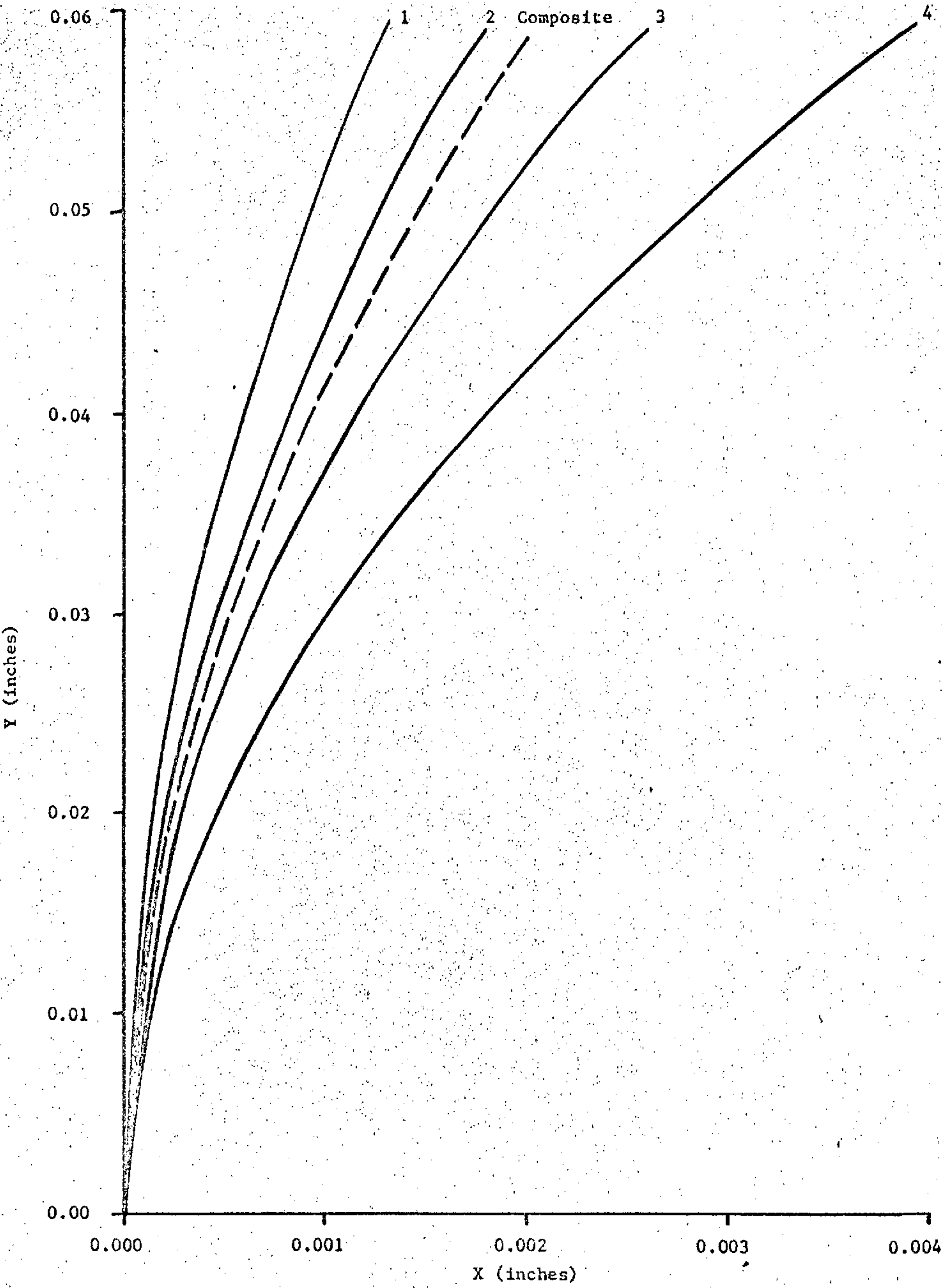
Maximum and rms spot diameters in the plane of best on-axis focus as a function of off-axis angle for a point source at a finite distance of 1000 feet from the LOXT. The plane of best on-axis focus is x = -1.53429 inches for the composite mirror system.

NOTE: 190 rays were traced through each individual mirror to get the spot sizes listed below. 760 rays were traced to obtain the data for the composite mirror system.

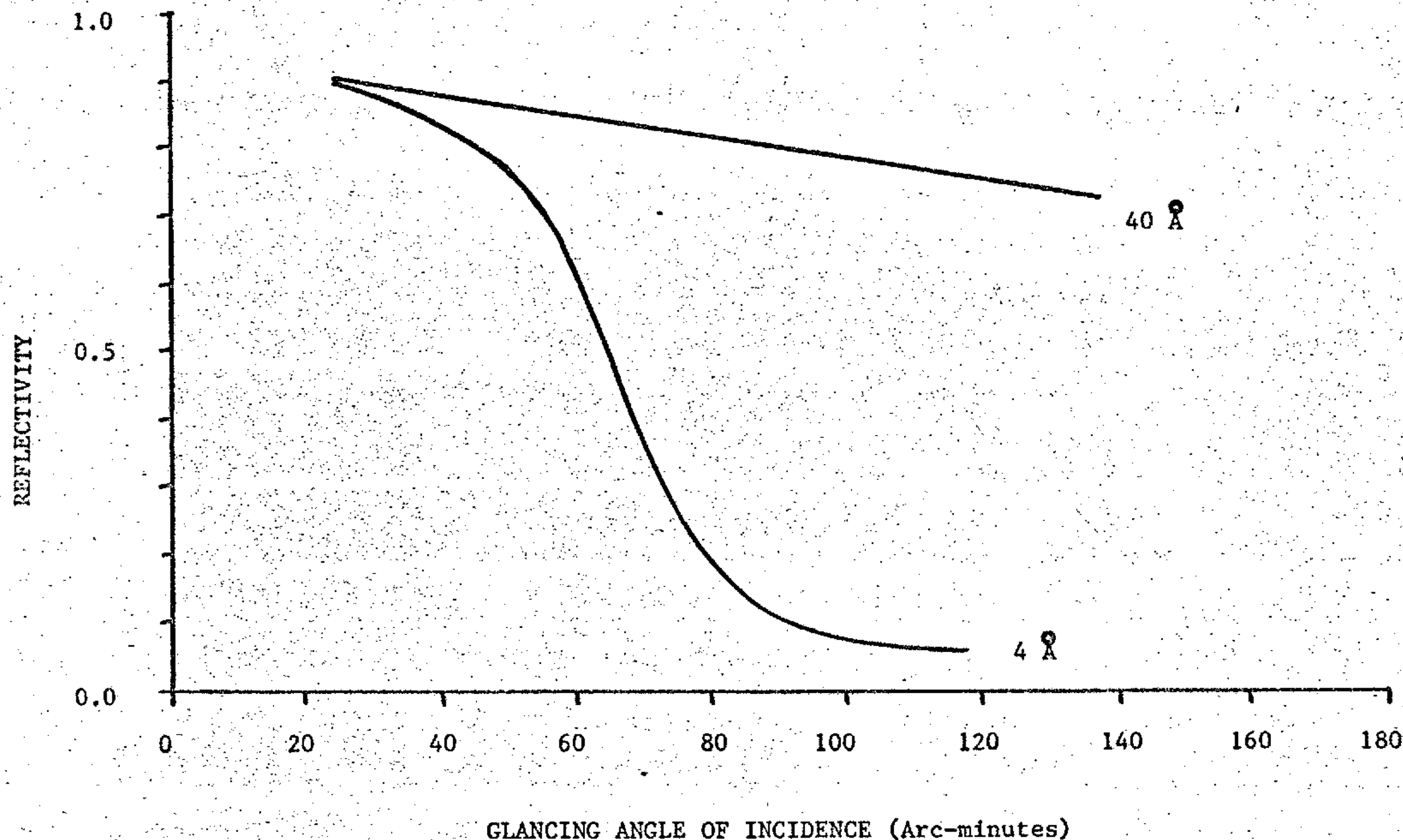
OFF-AXIS ANGLE (arc-minutes)	MIRROR NO.	RMS SPOT DIAMETER (arc-seconds)	MAXIMUM SPOT DIAMETER (arc-seconds)
0.0	1	1.137369	2.042655
	2	0.791560	1.317691
	3	0.795116	1.524142
	4	0.821337	1.509541
	Comp.	0.898190	2.042655
0.5	1	1.160135	2.307675
	2	0.867065	2.054933
	3	0.904466	2.303139
	4	0.954059	2.333116
	Comp.	0.981017	2.428256
1.0	1	1.237883	2.545708
	2	1.080768	2.986181
	3	1.197364	3.314195
	4	1.305846	3.442835
	Comp.	1.218110	3.635140
1.5	1	1.393410	3.397412
	2	1.413203	4.114219
	3	1.568367	4.477640
	4	1.704656	4.397936
	Comp.	1.537094	4.583448

OFF-AXIS ANGLE (arc-minutes)	MIRROR NO.	RMS SPOT DIAMETER (arc-seconds)	X-Coordinate of PLANE OF BEST FOCUS (inches)
0.25	1	0.0524019	0.00003
	2	0.0385480	0.00005
	3	0.0276298	0.00007
	4	0.0197450	0.00011
	Comp.	0.0537278	0.00005
0.50	1	0.106515	0.00014
	2	0.0799889	0.00019
	3	0.0605709	0.00028
	4	0.0495623	0.00043
	Comp.	0.111681	0.00022
0.75	1	0.163495	0.00032
	2	0.126384	0.00045
	3	0.102226	0.00064
	4	0.0938131	0.00098
	Comp.	0.177111	0.00050
1.00	1	0.224393	0.00058
	2	0.179394	0.00080
	3	0.154674	0.00115
	4	0.153955	0.00175
	Comp.	0.252615	0.00090
1.25	1	0.290131	0.00091
	2	0.240288	0.00125
	3	0.219079	0.00180
	4	0.231553	0.00273
	Comp.	0.340191	0.00142
1.50	1	0.361512	0.00131
	2	0.309976	0.00181
	3	0.296068	0.00260
	4	0.325193	0.00394
	Comp.	0.441103	0.00205

Plane of best focus as a function of off-axis angle for a point source at infinity.



Surfaces of best focus for a point source at infinity.



Reflectivity versus glancing angle of incidence for various x-ray wavelengths.

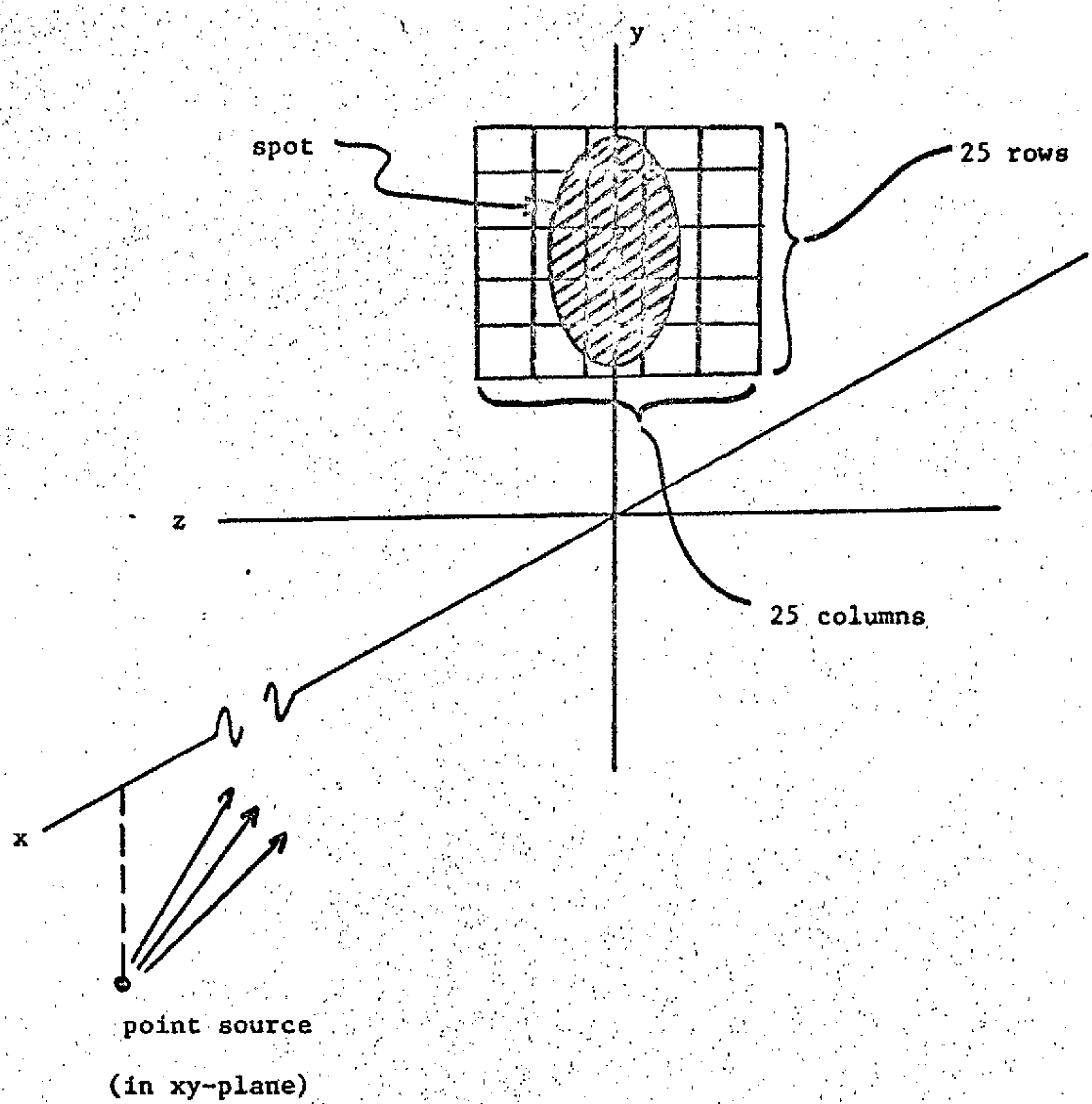
(Numerical data for these curves was taken from the reflectivity curves for nickel given in Fig. 2-2, p. 2-6 of Reference 2.)

LINE SPREAD FUNCTIONS - COMPOSITE SYSTEM - SOURCE AT INFINITY

The line spread functions shown below were computed by the general method outlined in Ref. 1. A square grid with 25 columns and 25 rows was used to collect the rays in each case. Thus, each line spread function is plotted as a histogram 25 bars in length.

The source at infinity was taken in every case to lie in the xy-plane. Thus, the resulting spots are centered on the y-axis and are symmetrical about the xy-plane. Consequently, all line spread functions L(z) are symmetrical about their centers. Approximately 5000 rays were traced to obtain the line spread functions at each off-axis angle.

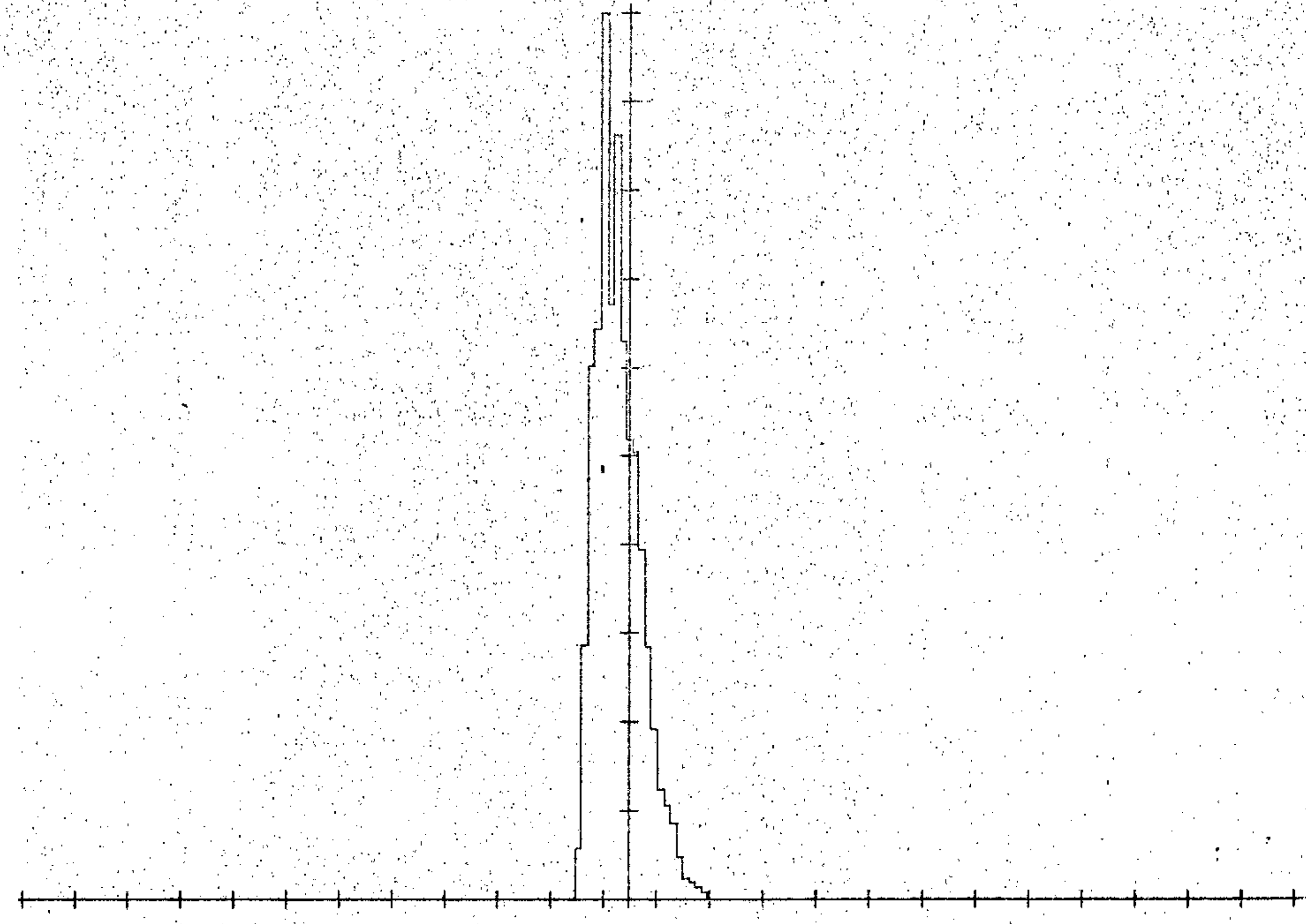
The line spread functions were calculated for $\lambda = 40$ Å and $\lambda = 4$ Å by using the reflectivity curves shown in the graph on the previous page. The contribution from each ray was taken to be $R_p(\theta_p) \cdot R_h(\theta_h)$, where R_p is the reflectivity for the glancing angle θ_p at the parabolic mirror and R_h is the reflectivity for the glancing angle θ_h at the corresponding hyperbolic mirror. The line spread functions were also calculated assuming uniform reflectivity ($R = 1.0$) for comparison with the results at 40 Å and 4 Å.



The line spread function L(y) is obtained by summing contributions from all rays across each row.

The line spread function L(z) is obtained by summing contributions from all rays along each column.

All line spread functions are normalized to unity at their maxima. The units on the vertical axes in the line spread histograms which follow are thus 0.1, 0.2, ···, 1.0. Each scale division on the horizontal axes represents 0.1 arc-second.



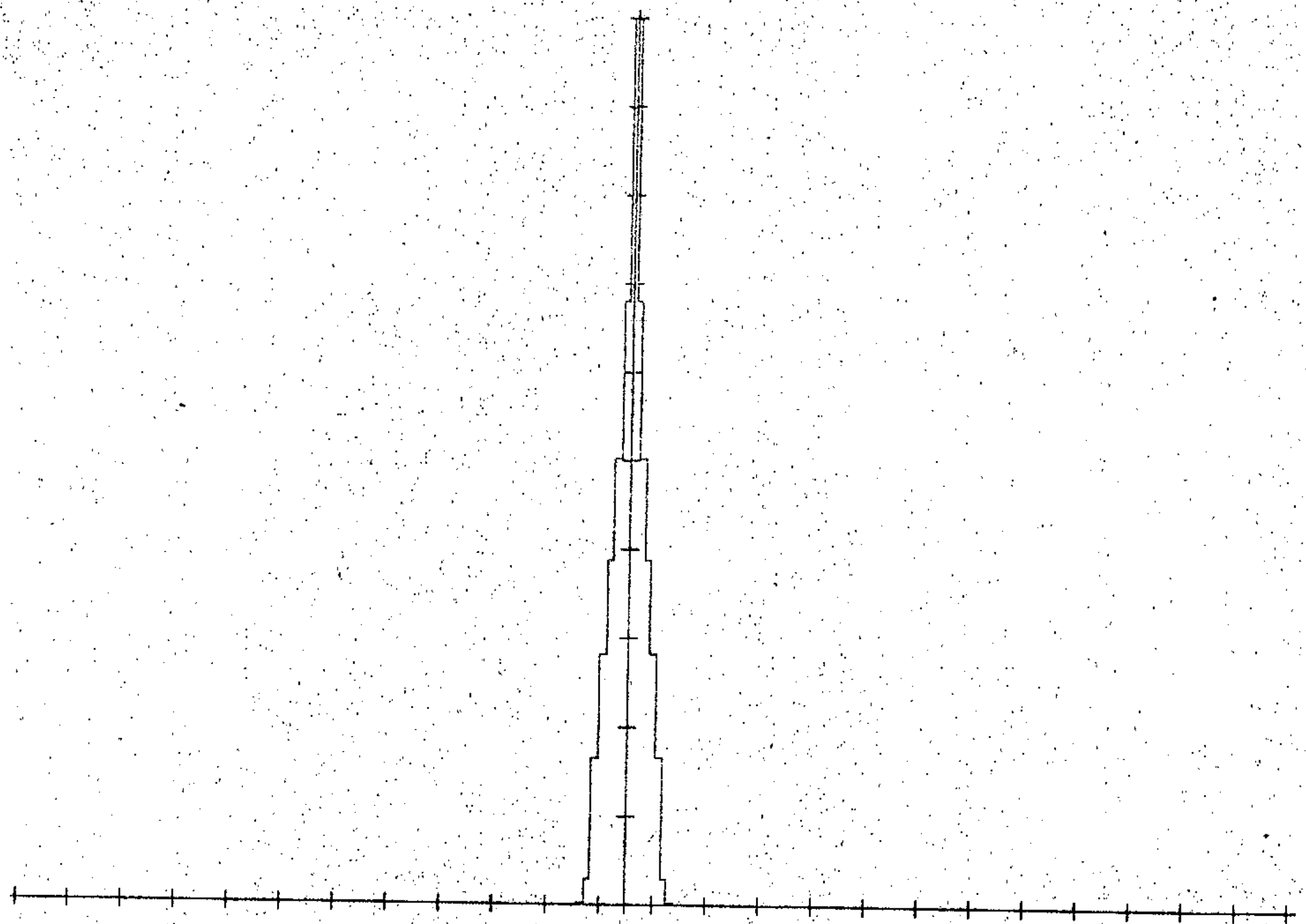
OFF-AXIS ANGLE = 0.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND

WAVELENGTH = 4 ANGSTROMS



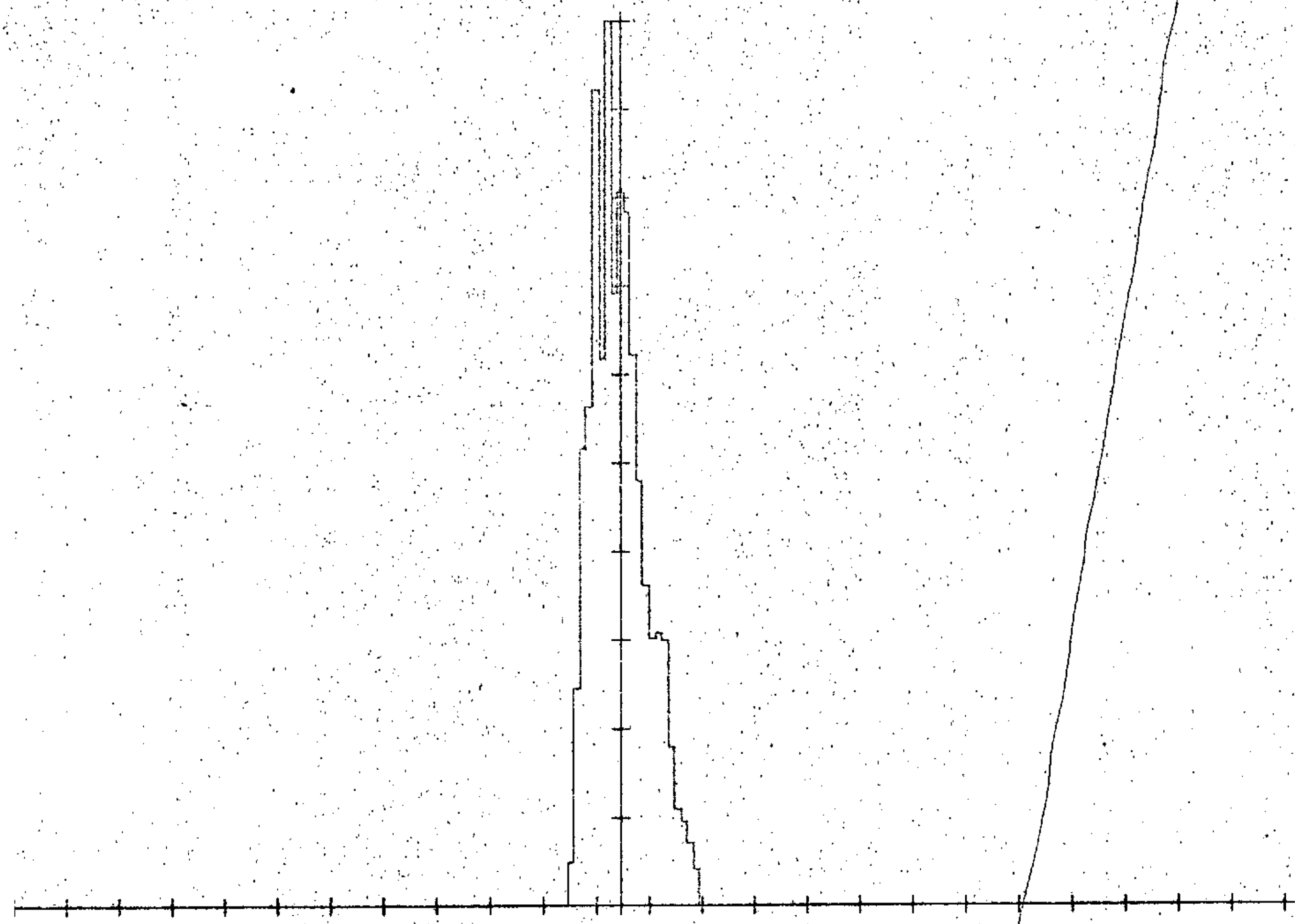
OFF-AXIS ANGLE = 0·5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 4 ANGSTROMS



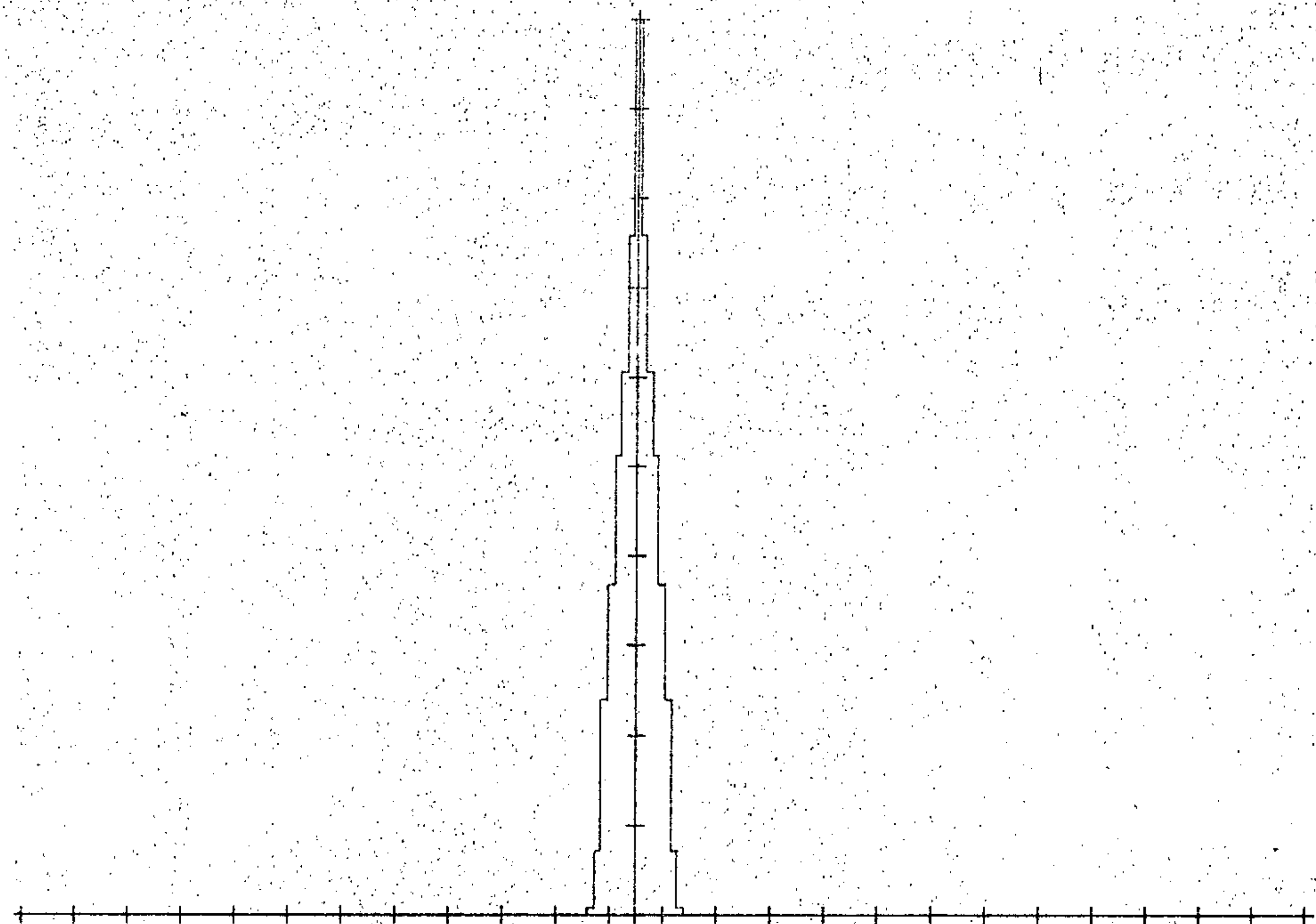
OFF-AXIS ANGLE = 0.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND

WAVELENGTH = 40 ANGSTROMS



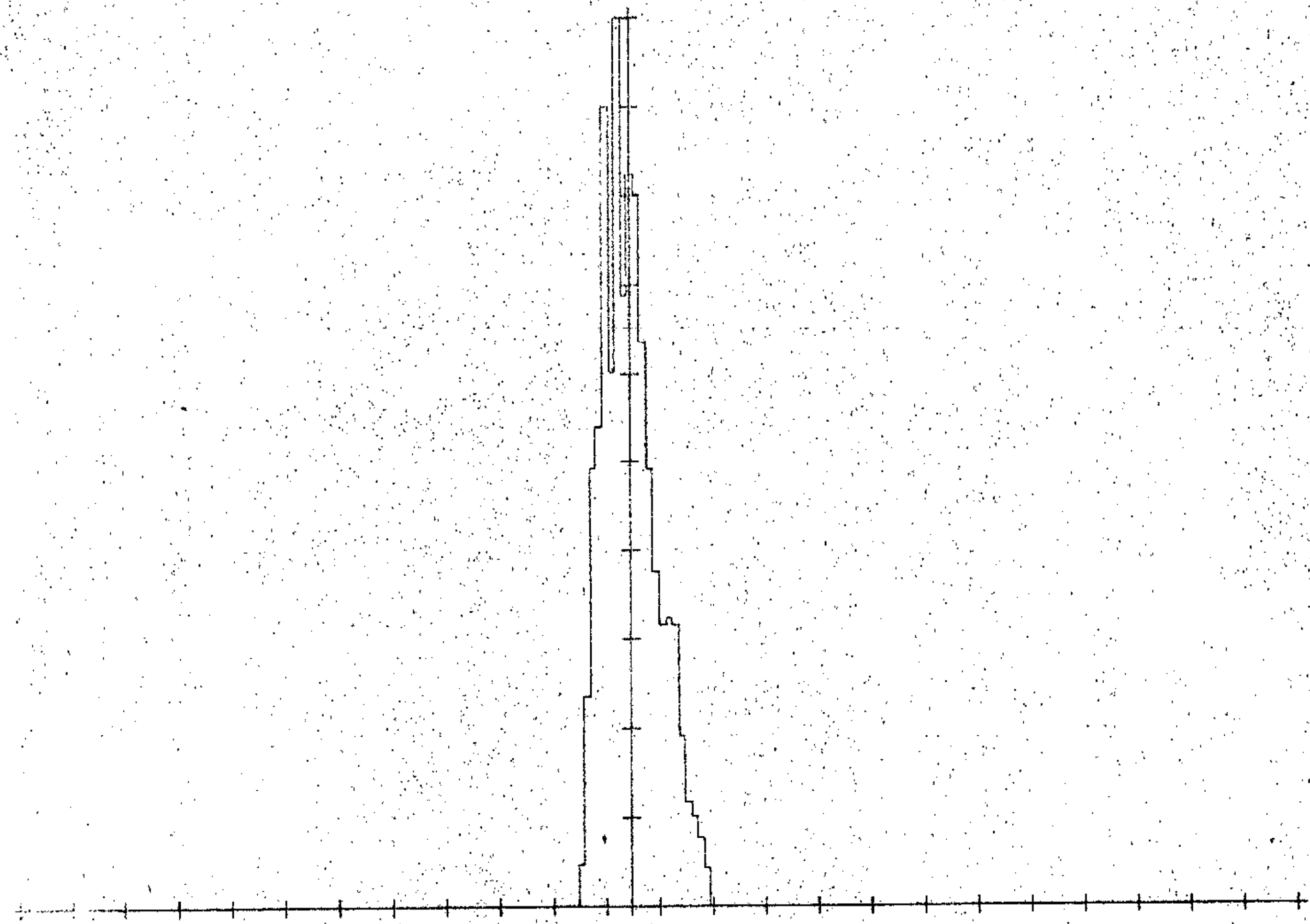
OFF-AXIS ANGLE = 0·5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 40 ANGSTROMS



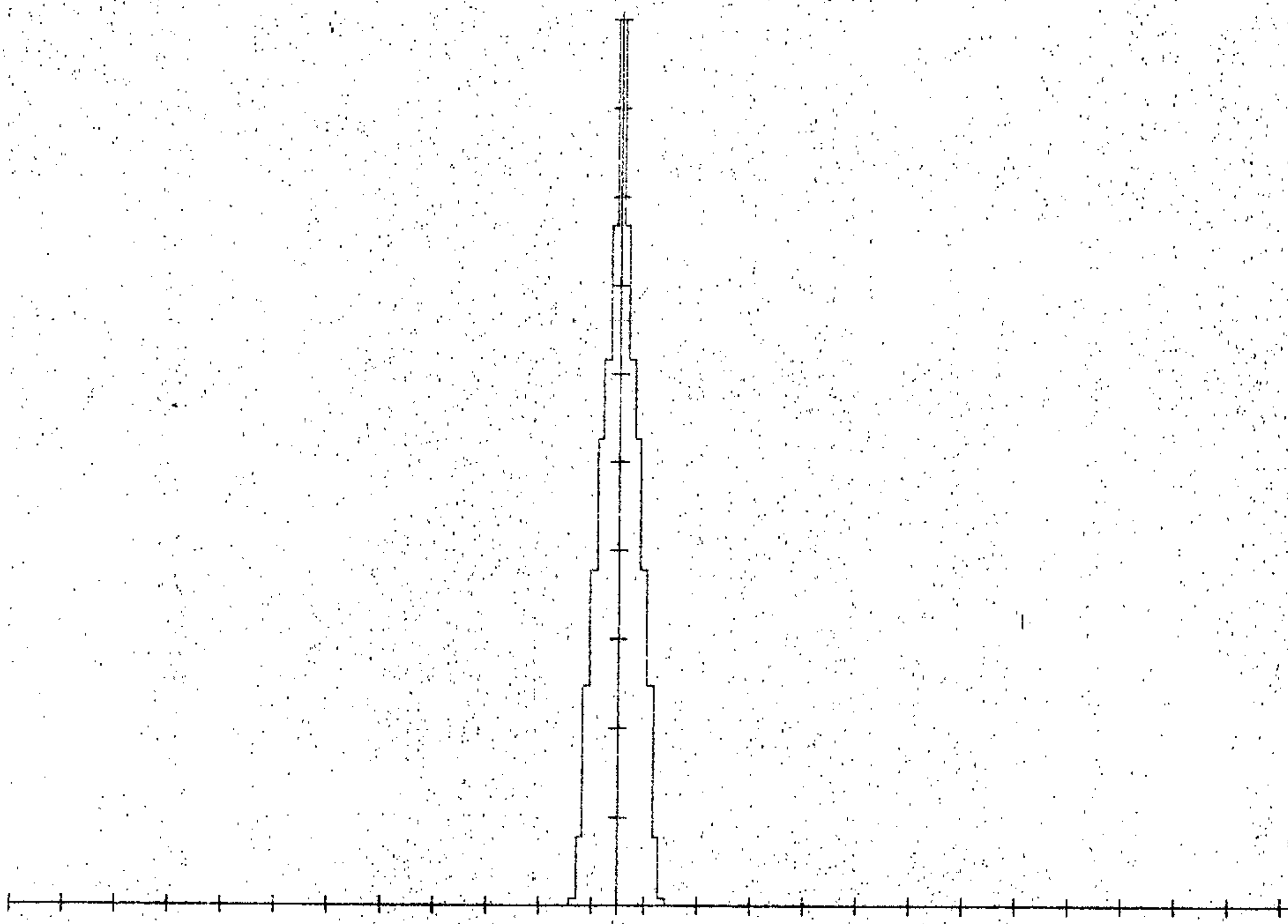
OFF-AXIS ANGLE = 0.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND

UNIFORM REFLECTIVITY (R=1.0)



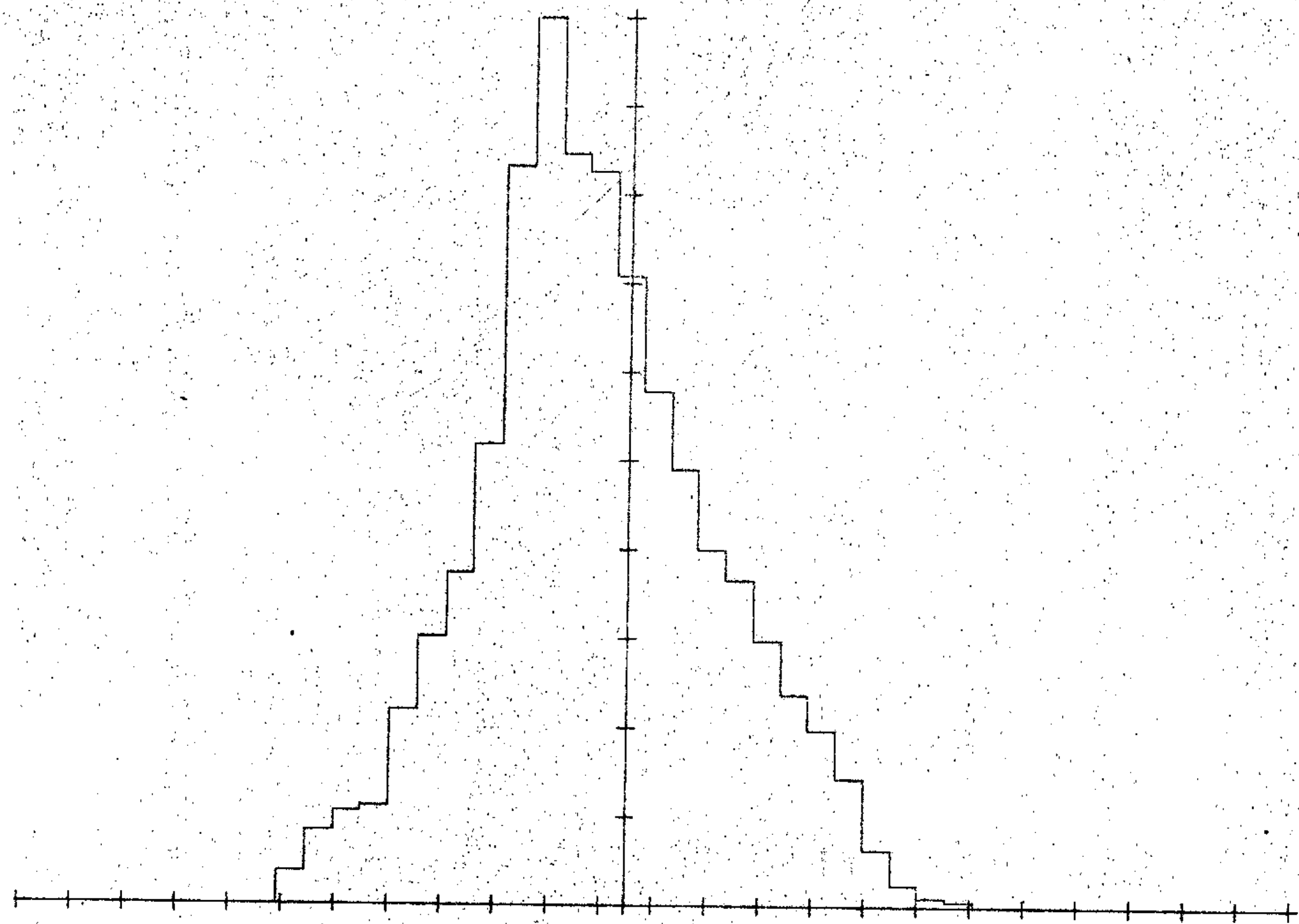
OFF-AXIS ANGLE = 0·5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0·1 ARC-SECOND

UNIFORM REFLECTIVITY (R=1·0)



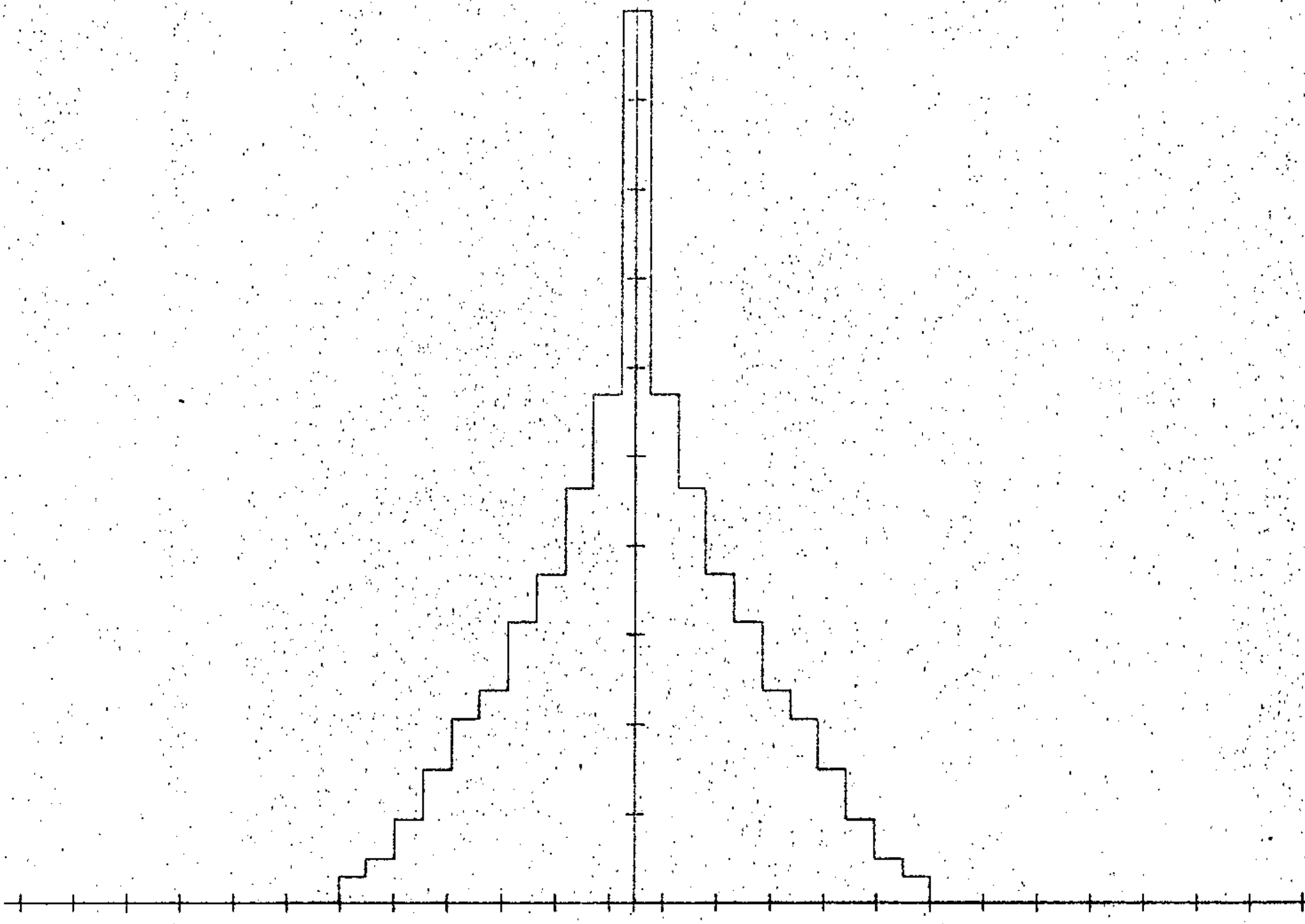
OFF-AXIS ANGLE = 1·5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 4 ANGSTROMS



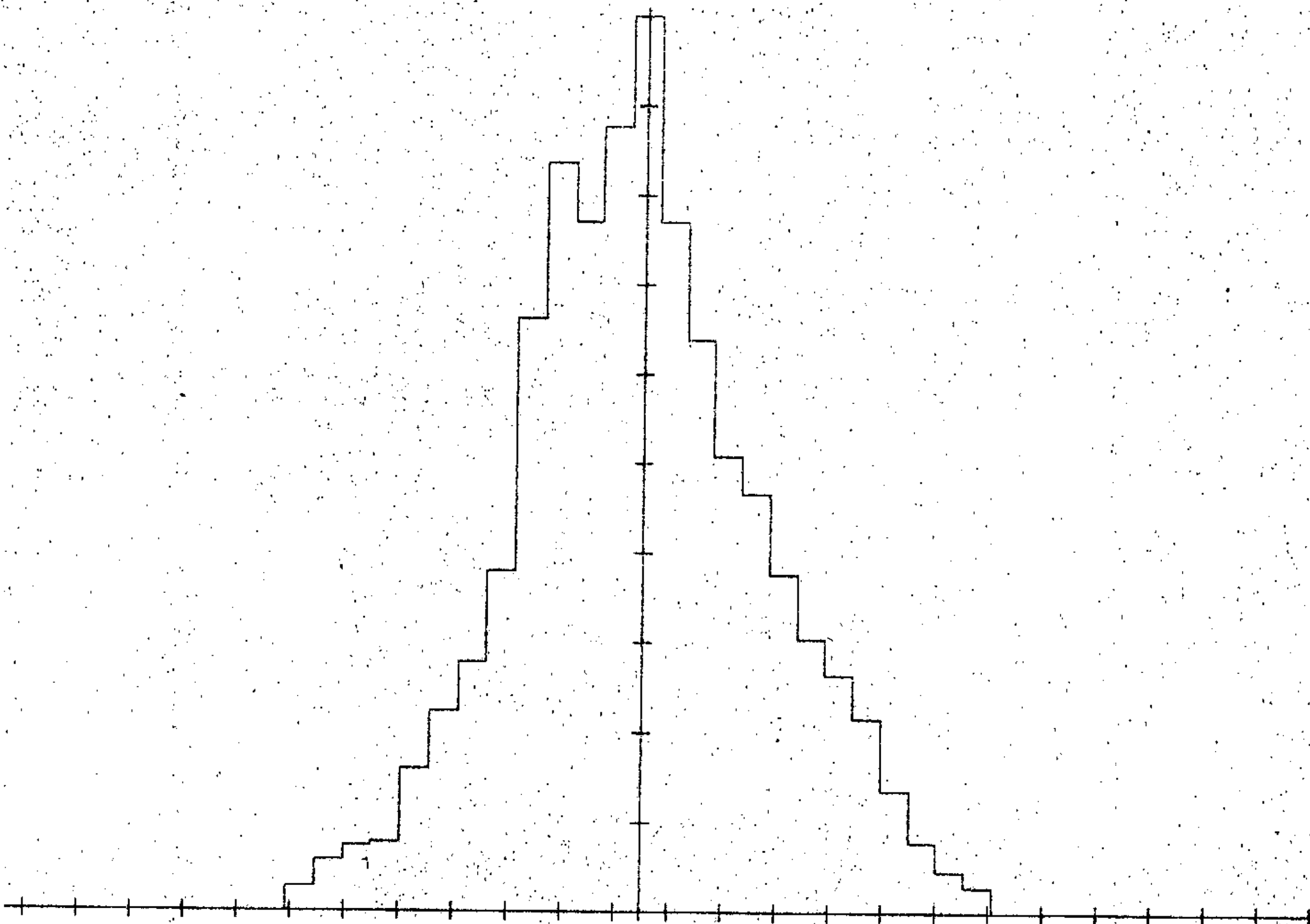
OFF-AXIS ANGLE = 1.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND

WAVELENGTH = 4 ANGSTROMS



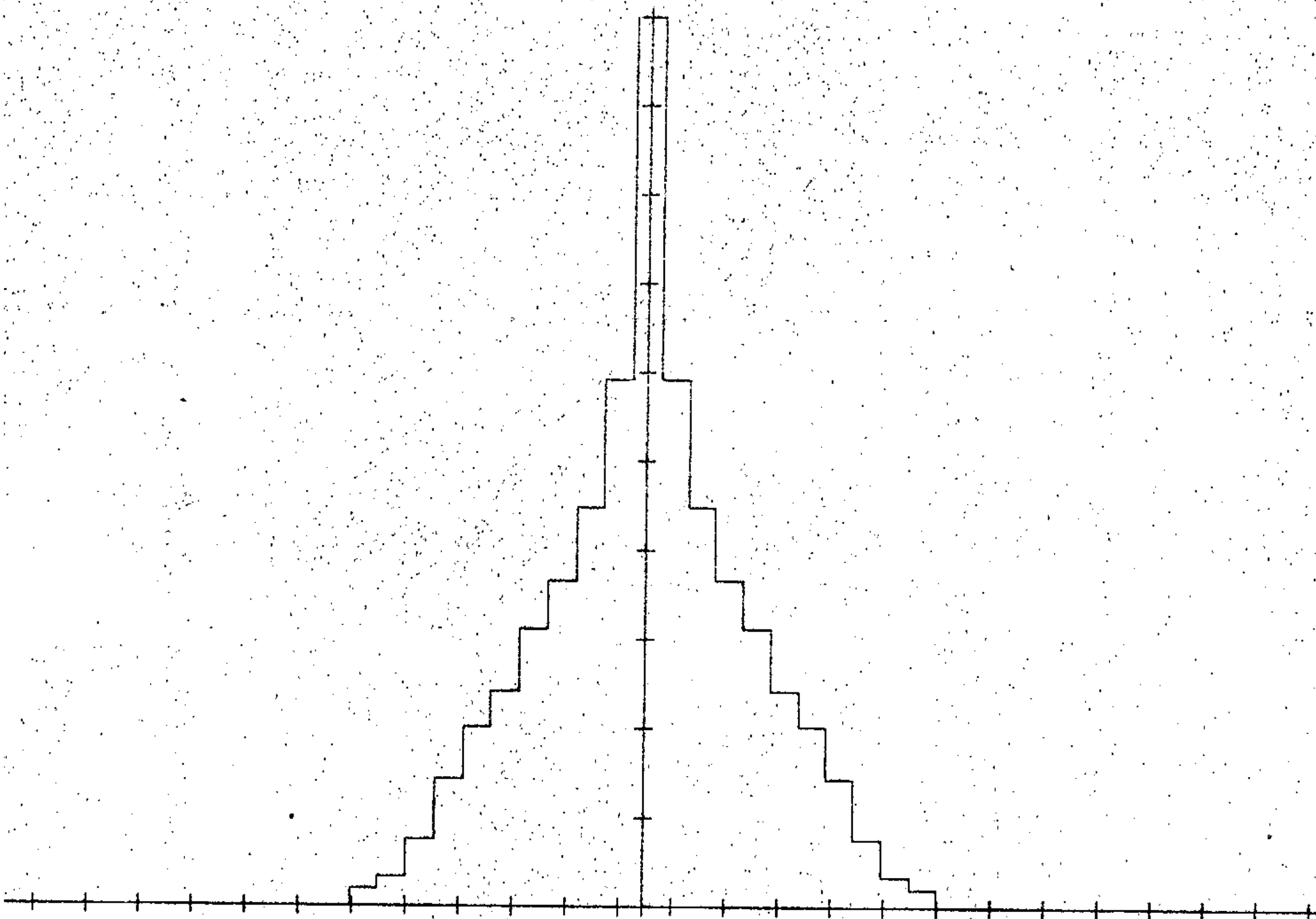
OFF-AXIS ANGLE = 1·5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 40 ANGSTROMS



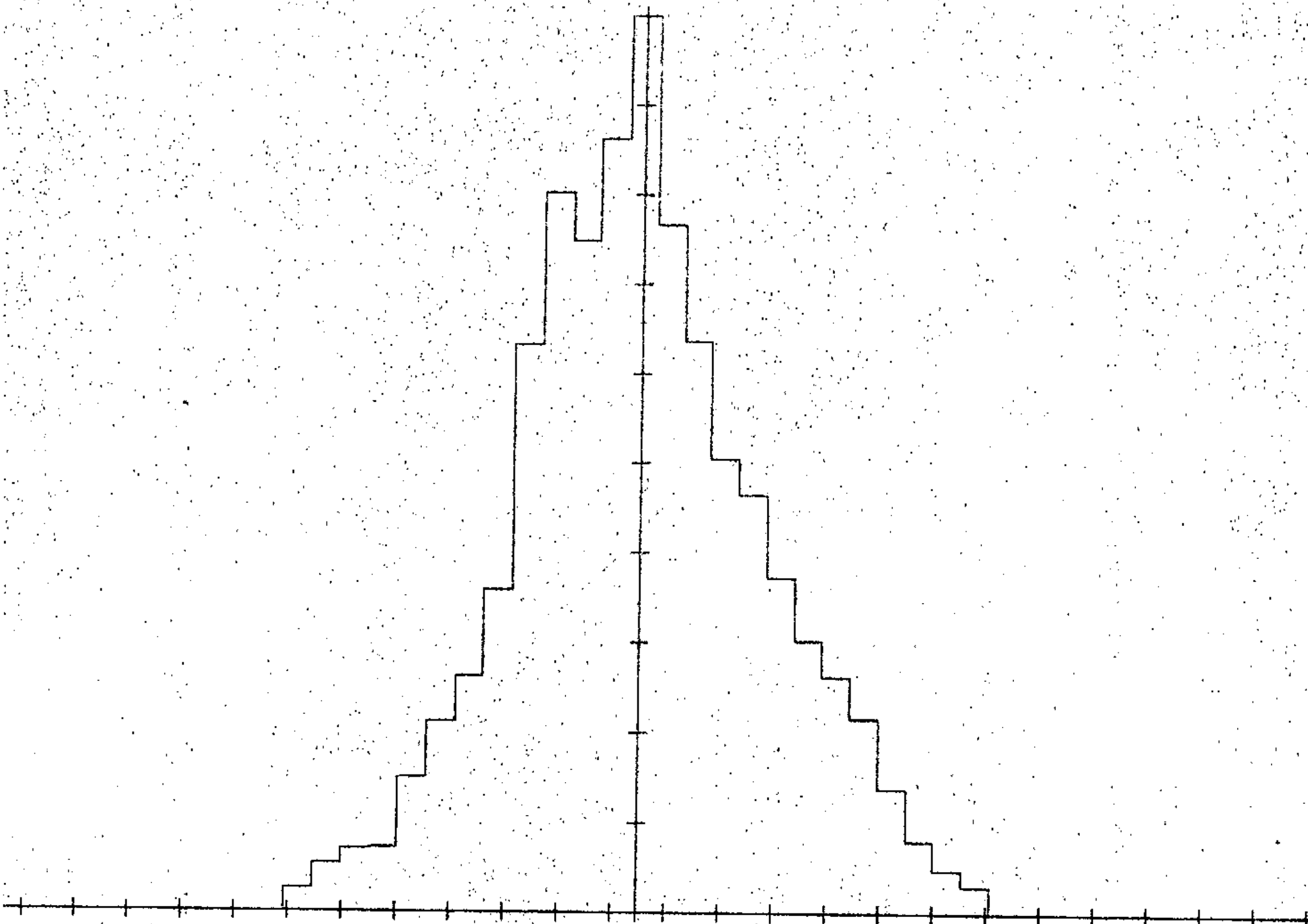
OFF-AXIS ANGLE = 1.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND

WAVELENGTH = 40 ANGSTROMS



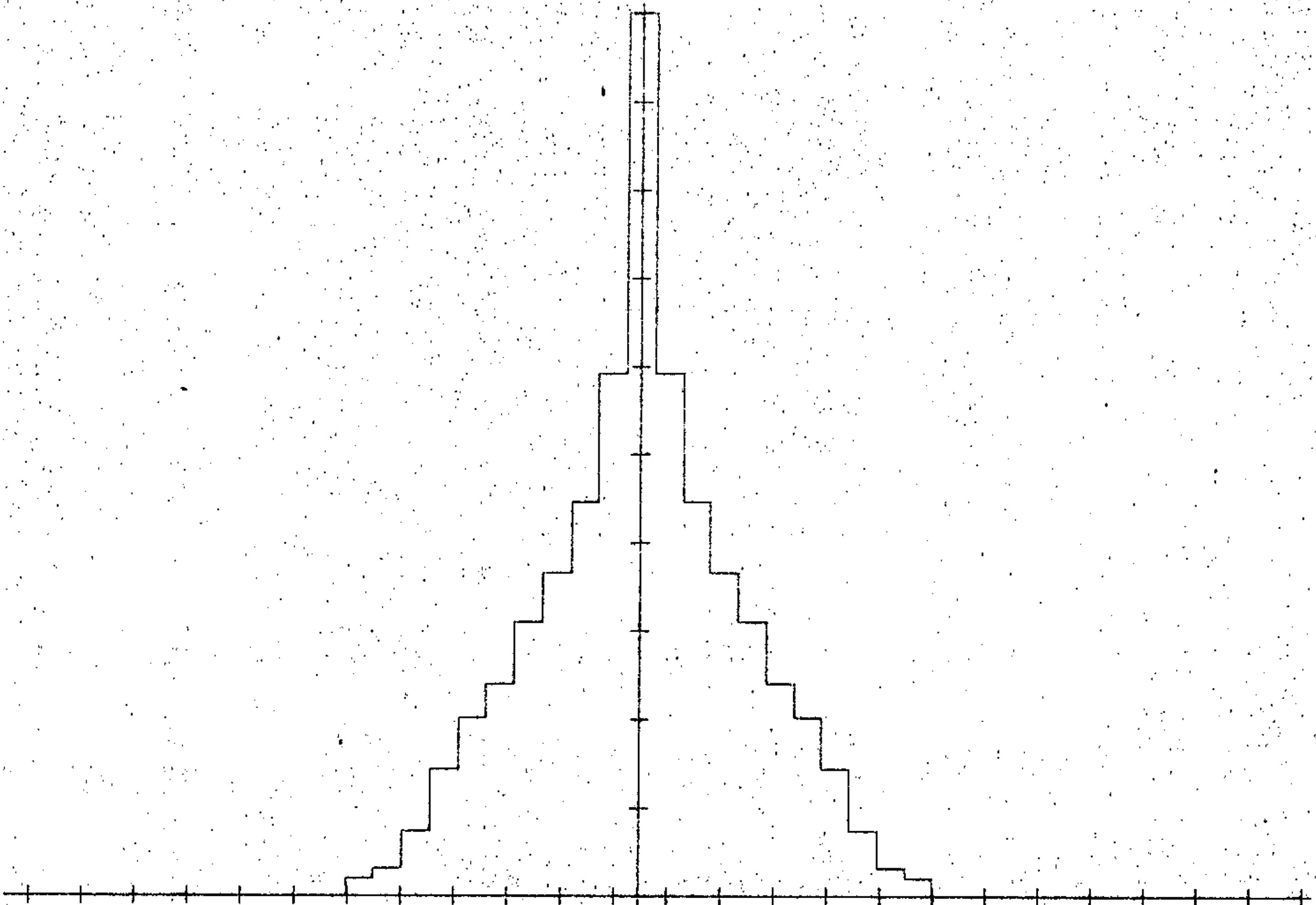
OFF-AXIS ANGLE = 1.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0.1 ARC-SECOND

UNIFORM REFLECTIVITY (R=1.0)



OFF-AXIS ANGLE = 1·5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

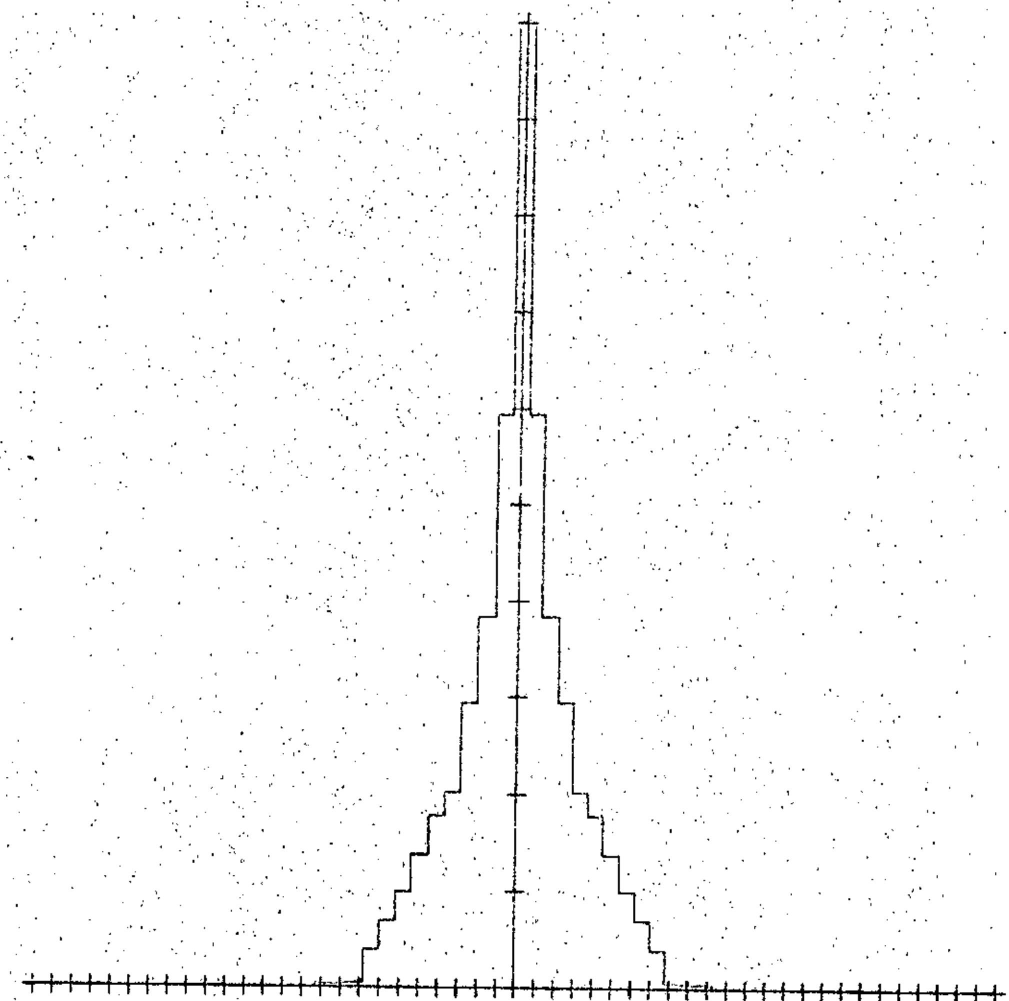
COMPOSITE MIRROR SYSTEM - SOURCE AT INFINITY

ONE SCALE DIVISION = 0·1 ARC-SECOND

UNIFORM REFLECTIVITY (R=1·0)

LINE SPREAD FUNCTIONS - COMPOSITE SYSTEM - FINITE SOURCE DISTANCE

The general procedure for calculating the line spread functions for a finite source distance of 1000 feet is the same as that used for a source at infinity.



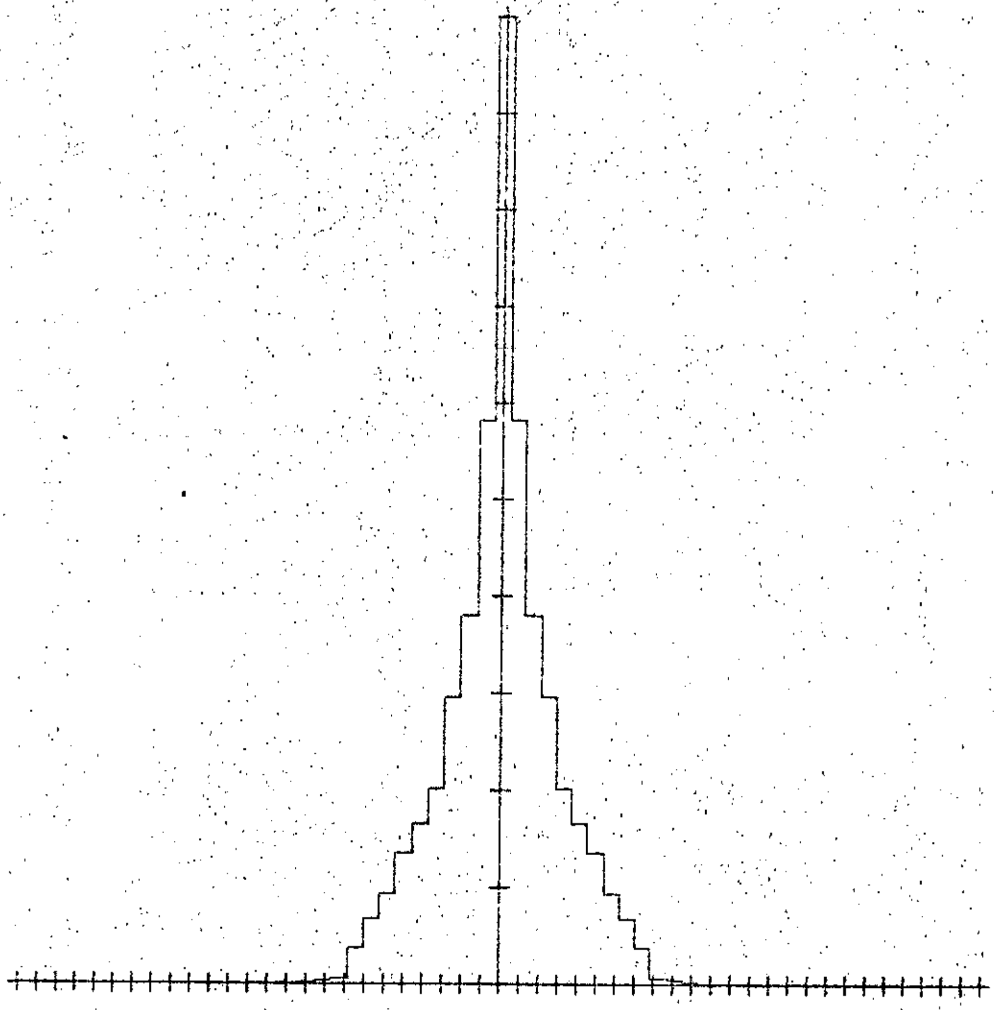
OFF-AXIS ANGLE = 0·0 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 4 ANGSTROMS



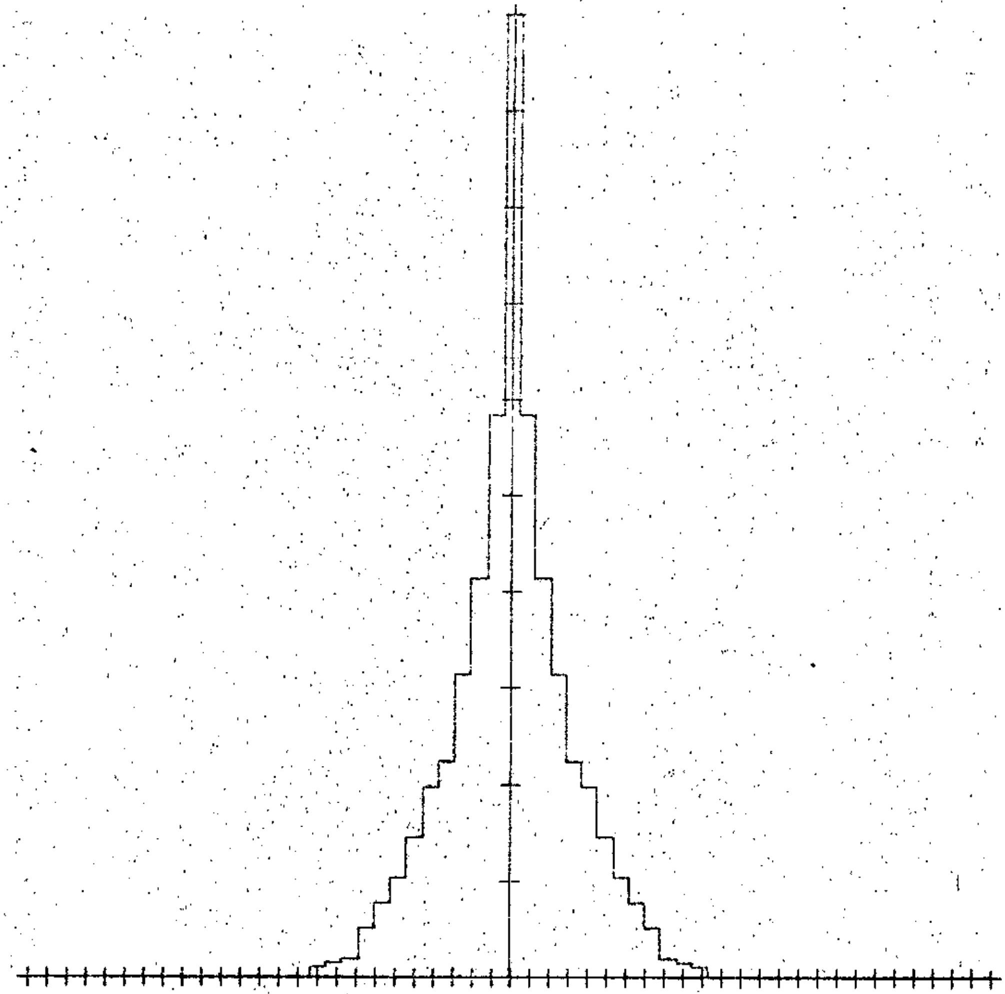
OFF-AXIS ANGLE = 0·0 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 4 ANGSTROMS



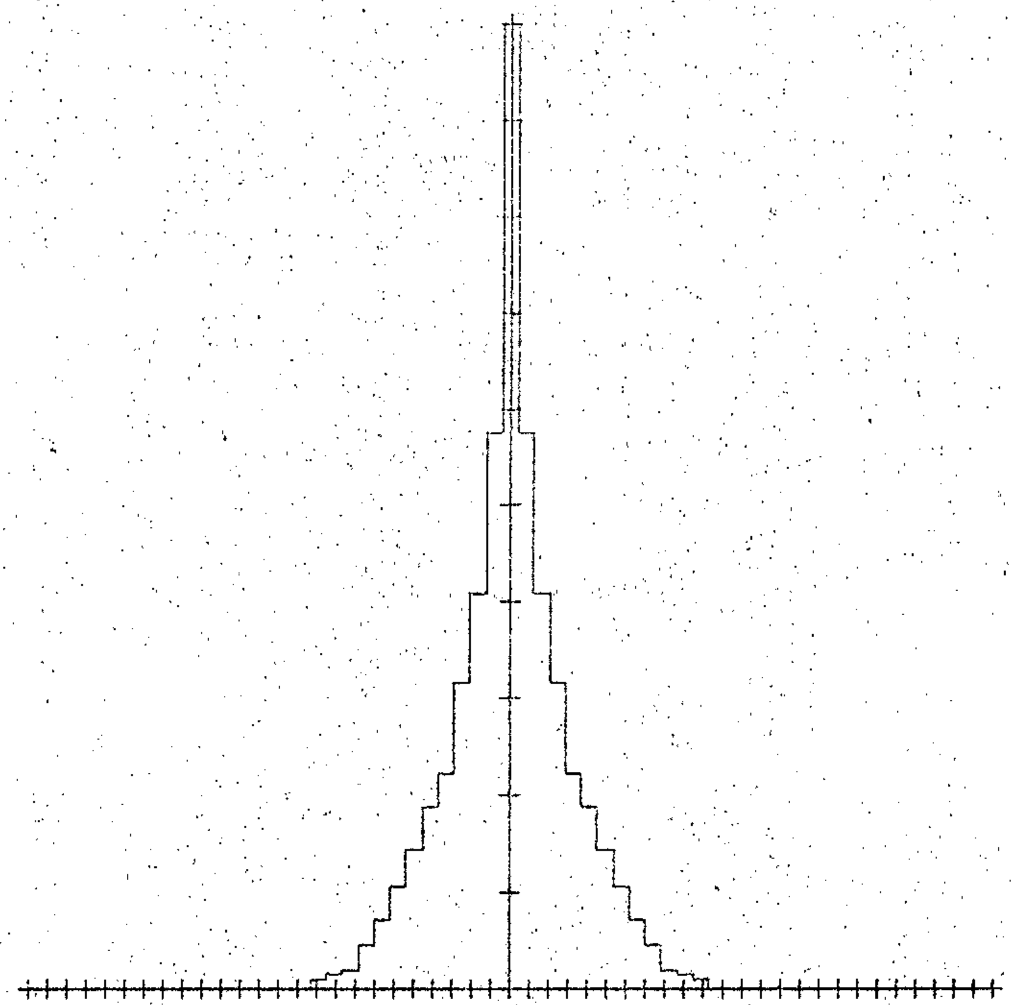
OFF-AXIS ANGLE = 0·0 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 40 ANGSTROMS



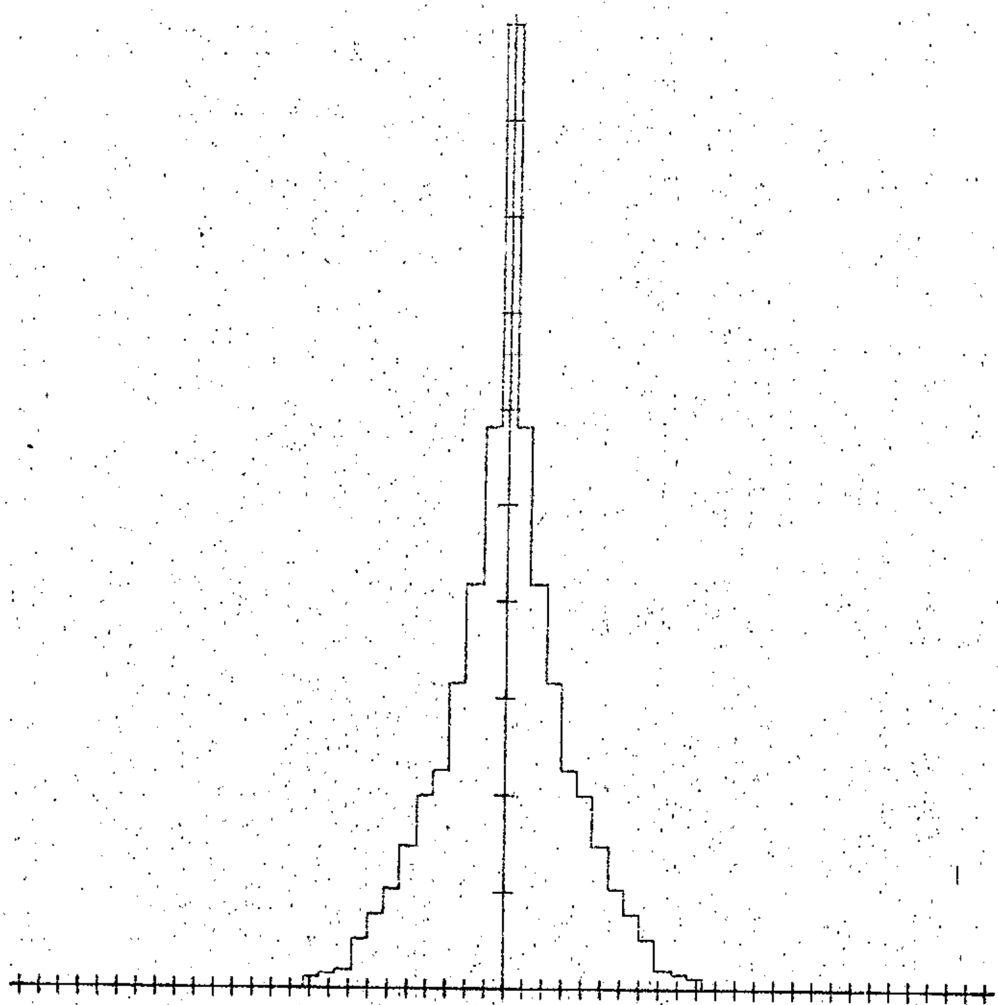
OFF-AXIS ANGLE = 0.0 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0.1 ARC-SECOND

WAVELENGTH = 40 ANGSTROMS



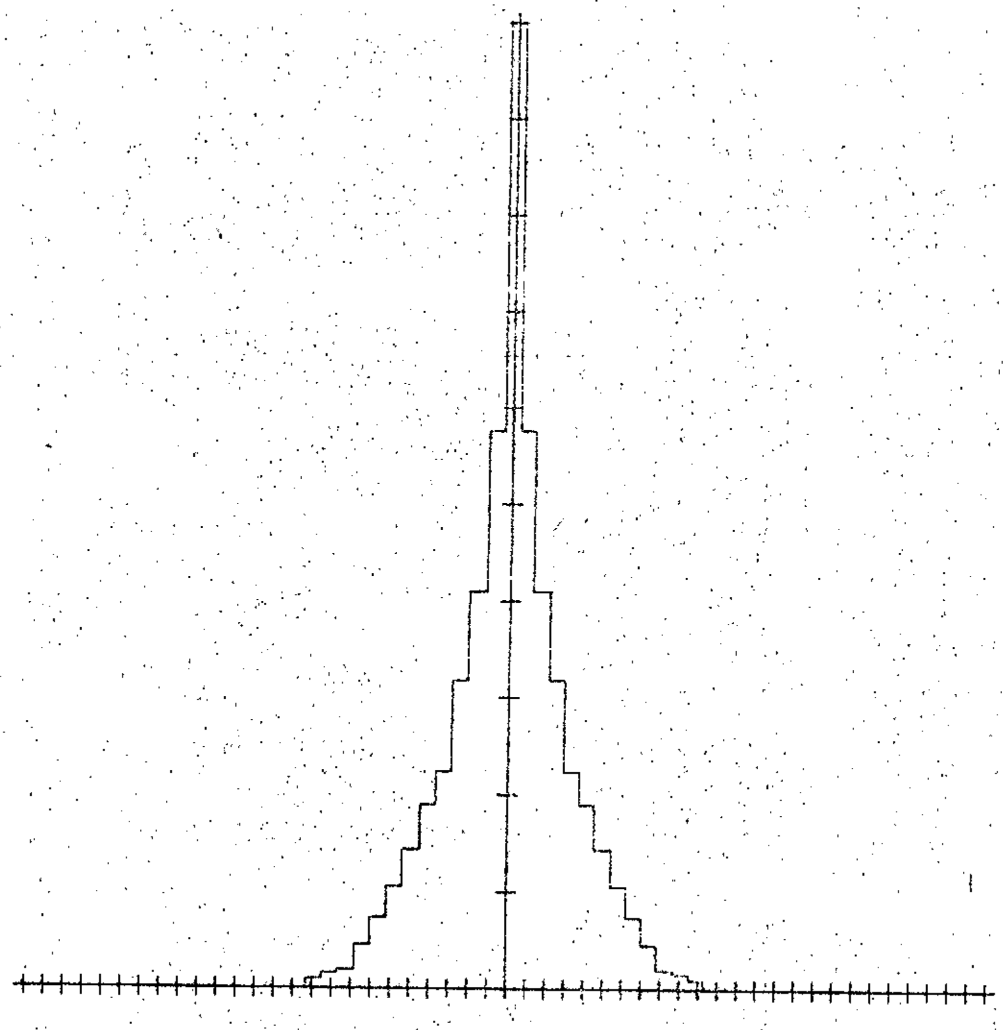
OFF-AXIS ANGLE = 0·0 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0·1 ARC-SECOND

UNIFORM REFLECTIVITY (R=1·0)



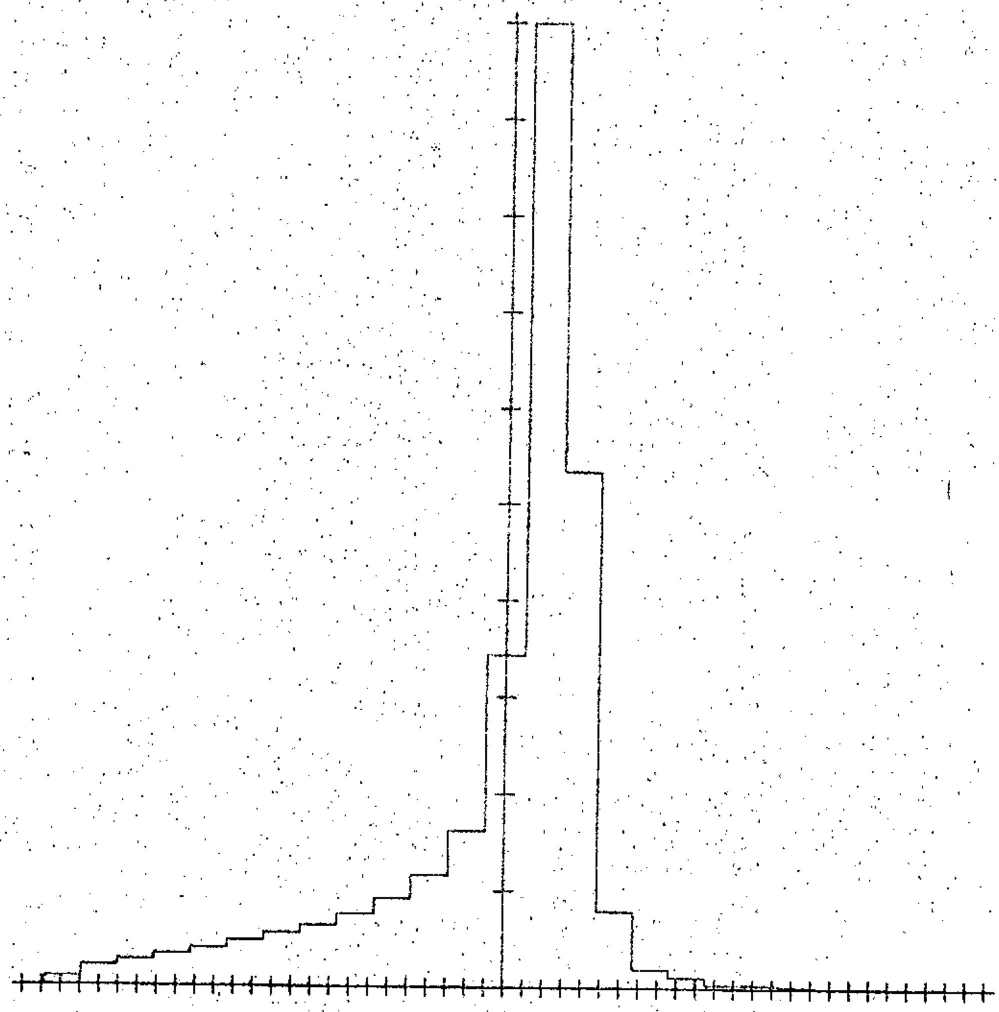
OFF-AXIS ANGLE = 0.0 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0.1 ARC-SECOND

UNIFORM REFLECTIVITY (R=1.0)



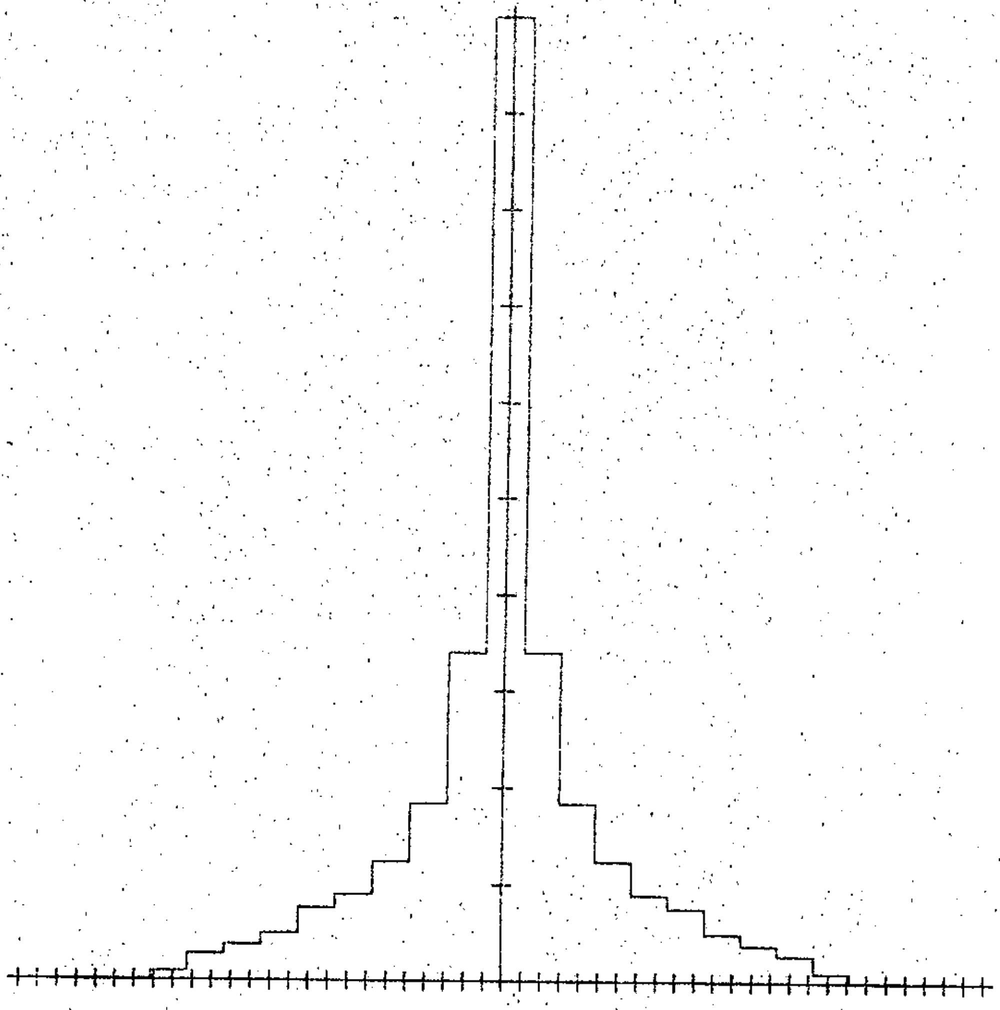
OFF-AXIS ANGLE = 1·5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 4 ANGSTROMS



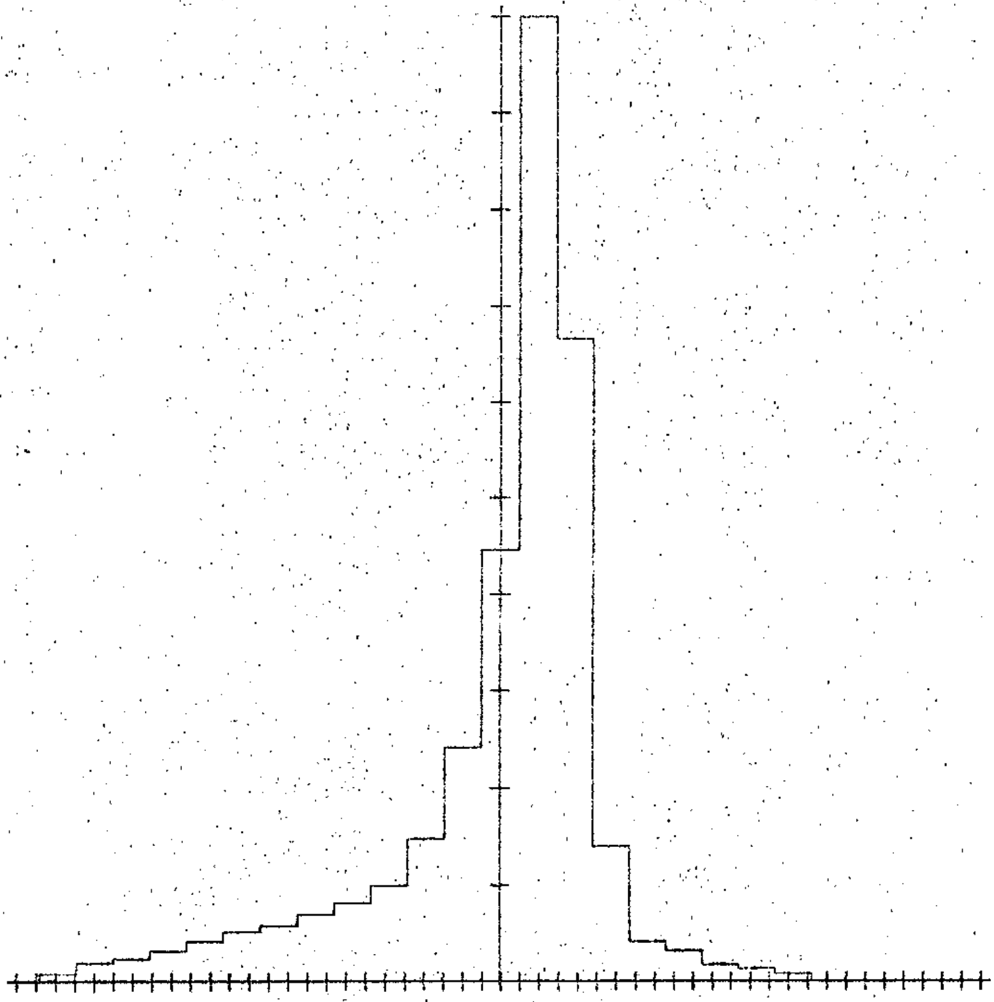
OFF-AXIS ANGLE = 1·5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0·1 ARC-SECOND

WAVELENGTH = 4 ANGSTROMS



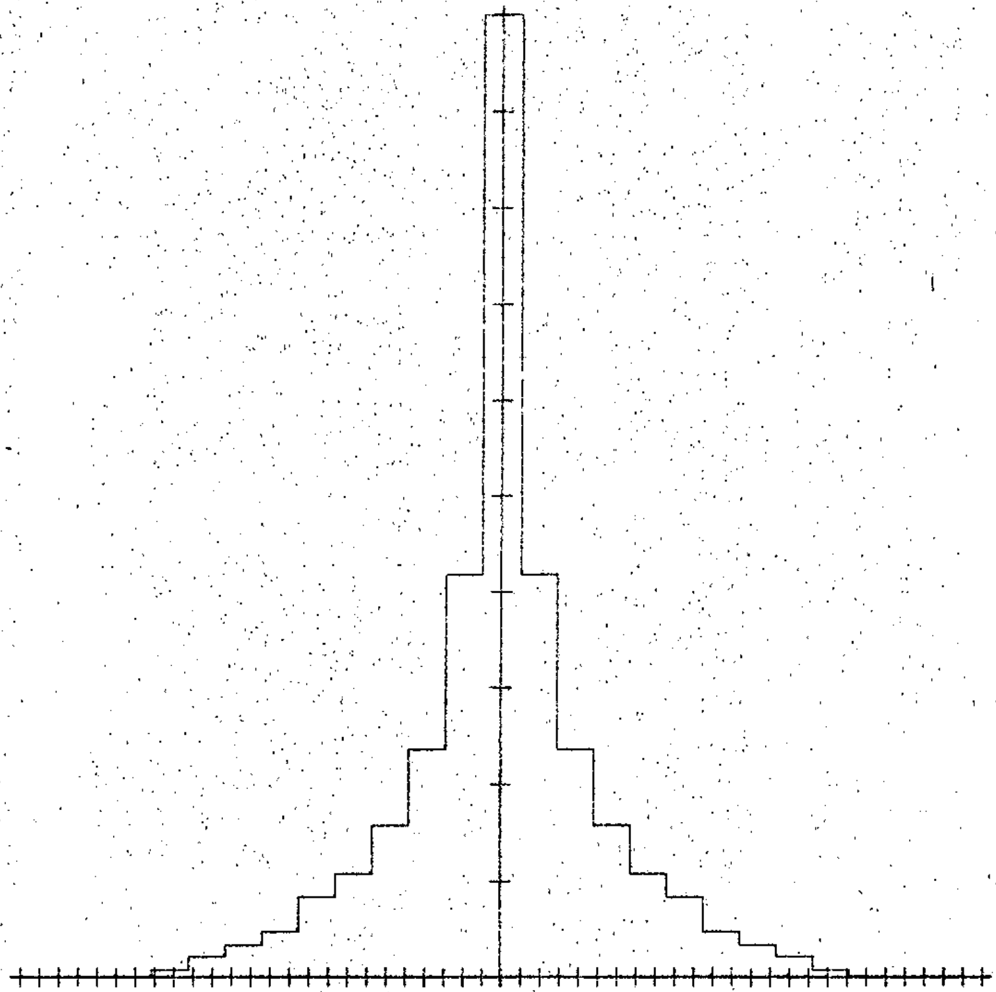
OFF-AXIS ANGLE = 1.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0.1 ARC-SECOND

WAVELENGTH = 40 ANGSTROMS



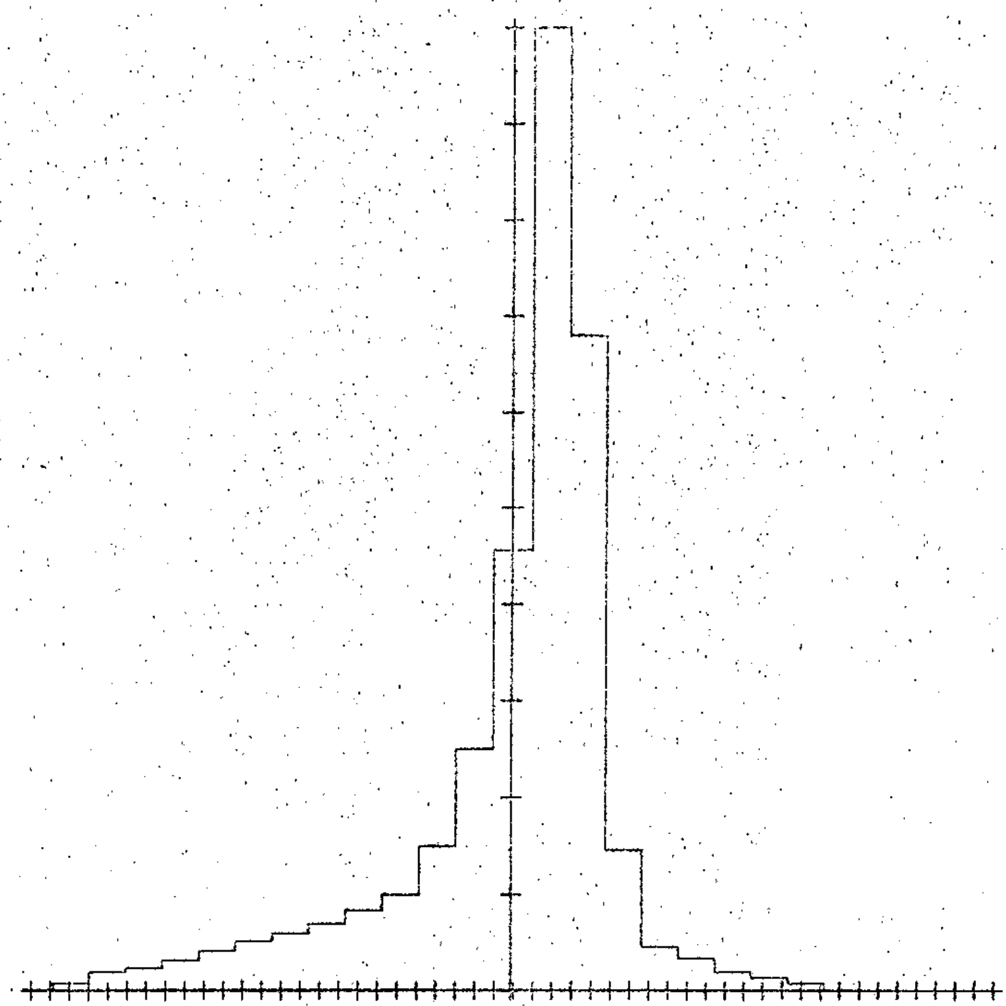
OFF-AXIS ANGLE = 1.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0.1 ARC-SECOND

WAVELENGTH = 40 ANGSTROMS



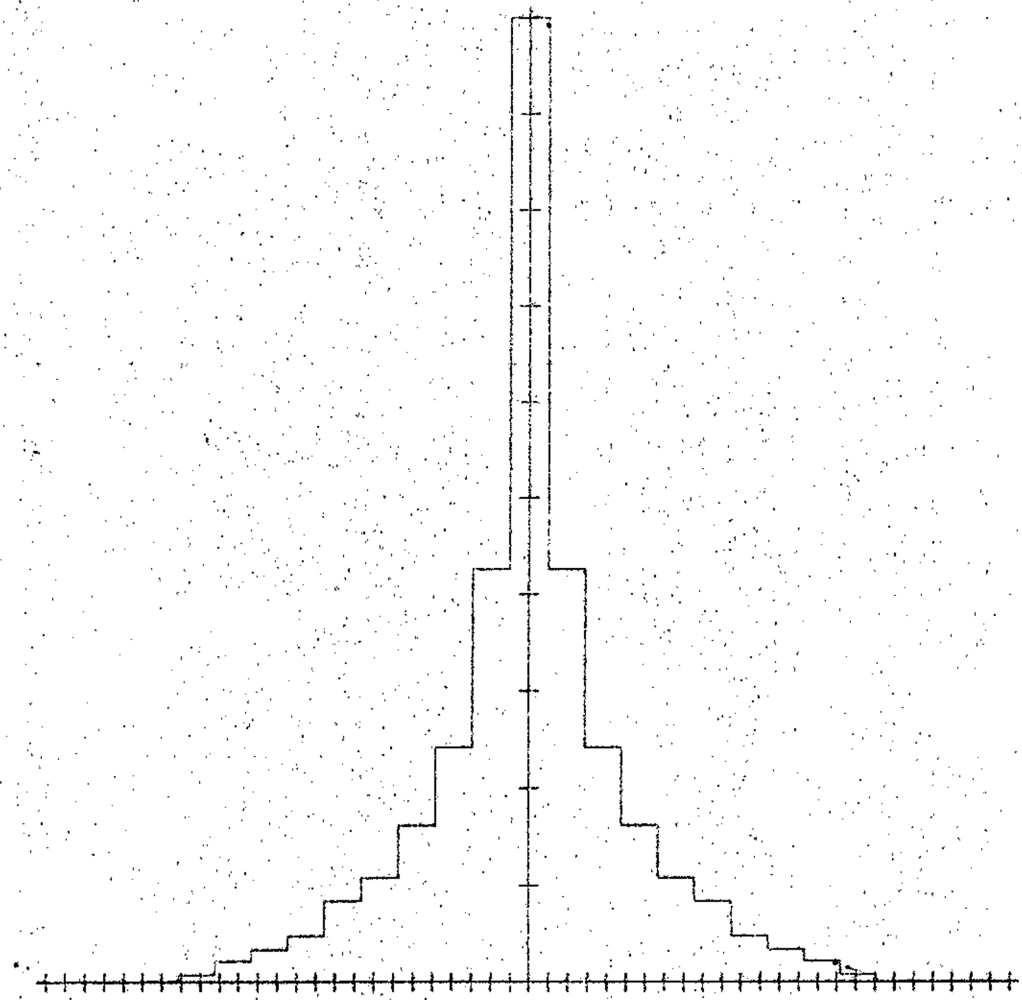
OFF-AXIS ANGLE = 1.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Y)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0.1 ARC-SECOND

UNIFORM REFLECTIVITY (R=1.0)



OFF-AXIS ANGLE = 1.5 ARC-MINUTES

LINE SPREAD FUNCTION L(Z)

COMPOSITE MIRROR SYSTEM - SOURCE AT FINITE DISTANCE

ONE SCALE DIVISION = 0.1 ARC-SECOND

UNIFORM REFLECTIVITY (R=1.0)

REFERENCES

1. J.W. Foreman, Jr., et al., Analytical Study of the Imaging Characteristics of the Goddard ATM X-Ray Telescope, Report No. SP-505-0279, Space Support Division, Sperry Rand Corp., Huntsville, Alabama (September, 1969).

2. L. VanSpeybroeck and R. Giacconi, Final Report: LOXT Mirror Design Study, Report No. ASE-3096, American Science and Engineering, Inc., Cambridge, Massachusetts (October, 1972).